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# A DETAILED CASE STUDY OF AN INTEGRATED REDUNDANT RELIABILITY MODEL USING THE PARALLEL-SERIES CONFIGURATION

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**ABSTRACT**

The Integrated Redundant Reliability Model (IRRM) is a new approach to reliability engineering that strengthens system dependability by using a Parallel-Series Configuration. The efficiency of the system is higher than that of a single-system factor with an equivalent configuration, but the performance of each component within the parallel-series structure matters. The research provides an Integrated Reliability Model (IRM) that considers impacts in each phase, component efficiency, and current restrictions, specifically designed for the parallel-series scenario. Thanks to redundant components arranged in parallel inside subsystems, this architecture provides instantaneous backup for a single-phase AC synchronous generator. The interconnected series structure ensures operational continuity even in the event of a subsystem failure, reducing vulnerabilities associated with both parallel and series setups. The integrated approach's objective is to raise dependability levels; it is particularly useful for critical systems. The model uses Lagrangean methods to compute variable quantities, effectiveness, and phase dependability, and it considers several elements to increase overall system efficiency. Changes made to Newton-Raphson methodology and simulation techniques to ensure integer outputs add to the realism of the values collected. This research provides significant new understandings into how integrated redundancy strategies could optimize system dependability and efficiency.

**Keywords:** *IRRR Model, Lagrangean Approach, Component Reliability, Newton-Raphson Approach, System Reliability*

## 1. INTRODUCTION

There are two main ways to increase a structure's inherent dependability: adding redundant units or incorporating components with higher inherent reliability. Using both strategies at once calls for more resources. This paper explores the optimization of structural reliability under various resource limitations, including load availability, size, and value. Generally, intrinsic value is the main factor used to evaluate reliability. Real-world situations, however, show that frequently invisible elements like size and load have a significant impact on and raise structural reliability.

A mathematical model for maximizing system dependability under linear constraints is presented by Mishra in 1972 [1], particularly for series-parallel systems with parallel redundancy at each step. In Part I, it is suggested to convert restricted optimization models into saddle point problems using Lagrange multipliers, derive maximum reliability requirements, and solve the resulting equations using Newton's method. The paper makes additional recommendations for enhancing larger systems' computational efficiency. Part II explores applying the Maximum principle to convert optimization models into multistage decision procedures that take design alternatives into account while remaining straightforward, realistic, and computationally efficient. A novel technique for utilizing zero-one integer programming to solve redundancy optimization problems Mishra, in 1971[2]. The method makes use of a Lawler and Bell algorithm that may be modified to accommodate other objective functions and constraints. Additionally, the paper discusses three problem variations, highlighting the ease of formulation and computational efficiency on digital computers, as emphasized by Mishra.

Exploring reliability evaluation, (Agarwal et al., 1975) emphasize its significance for engineers, comparing techniques for assessing system reliability [3]. They highlight challenges posed by non-series parallel systems and provide a systematic comparison of methods, illustrating their computational burden and reliability expression sizes through an example, aiding in methodological analysis. Devising an algorithm to minimize costs in reliable systems, (Agarwal and Gupta, 1975) simplified reliability expressions for general networks by identifying and modifying success paths to ensure their mutual disjointedness [4]. For reliability engineers working with large-scale systems, this method makes reliability studies more practical by increasing computational efficiency and decreasing their complexity. Investigating optimal allocation of multistate components to k series systems to maximize performance metrics, majorization and Schur functions are employed as key mathematical tools by (El-Neweihi et al., 1984), leveraging prior research on optimal component allocation [5]. Additionally, they illustrate the practical implications of their findings in enhancing reliability theory Presenting a heuristic approach for optimizing component assignment in parallel-series networks to enhance system reliability, (Prasad et al., 1991) address the challenge of selecting and assigning components to network positions, considering reliability impacts [6]. Their heuristic method efficiently solves this complex problem through a series of classical assignment problems. Computational experiments validate the effectiveness of the approach, yielding exact solutions in the majority of cases and providing close approximations when exact solutions are not feasible. Proposing a heuristic for optimizing series-parallel system assembly, (Baxter and Harche, 1992) establish its asymptotic optimality through probabilistic analysis [7]. Additionally, they offer bounds on the absolute and relative errors associated with any arbitrary heuristic used for the same purpose. In their study, (Fan and Wang, 1997) direct their attention to optimizing system reliability in parallel systems through variational techniques, with the goal of maximizing system profit by identifying optimal redundancy [8]. They introduce a simple computational procedure for efficiently designing multistage parallel systems, supported by detailed numerical examples in the paper that showcase the efficacy of their approach in achieving optimal system reliability.

Extending prior work, (Prasad and Ragawachari, 1998) enhance series-parallel system reliability through optimal allocation of interchangeable components [9]. Addressing NP-completeness in reliability optimization, they propose a practical procedure and develop an approximate linear programming model to minimize deviation. They also present an allocation refinement approach, which is backed by numerical studies showing encouraging results (Kuo and Prasad, 2000). Provided a summary, marked with annotations, of system reliability optimization techniques created after 1977 [10]. With applications to a broad range of design difficulties, they span heuristic, metaheuristic, exact, and multi-objective optimization in dependability systems. The study highlights the challenges of finding precise answers and the application of heuristic and metaheuristic algorithms to the problem of optimal redundancy allocation in this domain. Additionally, they offer an allocation refinement strategy that is supported by numerical analyses that yield positive outcomes (Kuo and Prasad, 2000). Gave an annotated summary of system reliability optimization methods developed since 1977. They cover heuristic, metaheuristic, precise, and multi-objective optimization in dependability systems, with applicability to a wide range of design challenges. The study draws attention to the difficulties in obtaining exact solutions and the use of heuristic and metaheuristic algorithms in solving the optimal redundancy allocation problem in this field.

By applying a parallel evolutionary approach to integrate the optimal redundancy solutions, the reliability-redundancy allocation problem (RRAP) is handled in a more sophisticated way (Ho-Gyun Kim and Chang-Ok Bae, 2006). Their approach takes into account both active and cold standby redundancy solutions to optimize system reliability [11]. Interestingly, they improve accuracy over earlier predictions by introducing an accurate reliability function for cold standby redundant subsystems. They underline how much better their strategy is than previous studies and provide numerical examples to show its efficiency. When looking into coherent redundant system dependability design and evaluation, consideration should be given to weight, load, quantity, size, space, volume, and weight in addition to cost. Their detailed analysis takes these unimportant elements into account, using the Lagrange multiplier for initial design and assessment. The approach produces well-known solutions for unit and phase reliabilities that may be used in real-world situations by combining integer and dynamic programming approaches, Sridhar et al., in 2021 [12]. Furthermore, the modeling performance using real datasets is illustrated by talking about parameter estimation techniques as the EM method and maximum likelihood.

Investigate efficiency models considering worth, load, and size constraints while optimizing parallel-series Integrated Reliability Model (IRM) systems (Akiri et al., 2022). They draw attention to the usefulness of these models in situations when system worth is low, especially for redundant systems configured in a parallel-series fashion [13]. The IRM provides a logical answer by using the Lagrangean approach to compute important parameters such as the number of factors, factor efficiencies, phase efficiencies, and system efficiency. Additional improvements made possible by dynamic programming guarantee real-world applications.

We analyze the evolution of k-out-of-n systems in a recent work, concentrating on redundant reliability systems created with the aid of heuristic programming (Srinivasa Rao et al., 2022). They work on nonlinear programming issues, particularly those involving integer variables in system reliability optimization [14]. This paper investigates specific architectures of the objective function and constraints to get exact answers for dependability optimization problems. The authors also suggest ways to improve system reliability optimization with redundancy and integrated reliability models through a review of the literature. Investigate the design of integrated redundant reliability systems for k-out-of-n configurations (Srinivasa Rao et al., 2022). They examine how structural reliability is impacted by factors other than cost, such as load, size, and volume. The paper provides answers for element, phase, and structure reliability using the Lagrange multiplier approach [15]. They also use a heuristic procedure to produce almost optimal integer answers, which are improved by the Dynamic programming technique, as illustrated by a numerical example.

Performed an analysis of the Integrated Redundant Reliability Model (IRM) using the k-out-of-n configuration. (Velampudi, Srinivasa Rao, et al., 2023). They suggest integrating several technologies into a single system in series, which is appropriate for k out of n systems and effectively achieves improved efficiency [16]. Using Lagrangean approaches, the authors determine optimal factors and phase reliabilities under constraints related to load, size, and cost. They use approaches from simulation and dynamic programming to further validate their findings. To improve coherent system reliability, two cold standby components are investigated (Roy and Gupta, 2024). They look into employing sequential standby activation to restart these kinds of systems, showing that starting with a more reliable standby component can extend the system lifetime [17]. The study provides numerical data by computing reliability functions and examining mean residual life functions, highlighting the significance of its findings. (Gregory Levitin et al., 2024) enhance 1-out-of-n standby system modeling by merging dynamic resource supply and storage units with activation moment-dependent component operational time limits [18]. Their model accounts for non-identical components whose activation and resource dynamics determine their operating periods. They improve the mission success probability (MSP) by improving the standby component activation sequencing using a novel numerical technique. A case study of a standby sensor system demonstrates the effectiveness of the model and offers suggestions for enhancing MSP.

Despite the significant progress made in optimizing system reliability and performance,

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there remains a notable research gap in systematically comparing and evaluating the effectiveness of various optimization techniques across different types of systems and constraints. While individual studies have contributed valuable insights, a comprehensive comparative analysis is lacking, hindering the development of standardized approaches for reliability optimization across diverse applications and industries. Closing this gap would provide invaluable guidance for reliability engineers in selecting the most suitable optimization methods for specific system configurations and constraints, ultimately advancing the field of reliability engineering.

An in-depth investigation into an over-reliability model, considering multiple constraints, was conducted to maximize the suggested configuration's efficiency. The current task investigates several unknowns at a certain point in time, including individual elements ($X_{cs}$), element reliability ($r_{ce}$), and stage reliability ($R_{pd}$). These considerations aim to mitigate several constraints, thus amplifying the structural reliability, leading to the development of a Unified Reliability Model (URM). Existing literature has shown that augmenting United Reliability Models involves introducing value constraints where a fixed relationship between value and reliability exists. In a novel approach, this study integrates load and size as supplementary constraints, in addition to value, to craft an improved redundant reliability system for structures following the parallel-series composition principle. The conclusions are discussed in section 6.

## 2. METHODOLOGY

### 2.1 Assumptions and Symbols:

- Homogeneity assumption: All components within each stage are considered identical in terms of reliability.
- Statistical independence assumption: Components are treated as statistically independent entities, implying that the failure of one element does not impact the performance of other elements in the system.

$R_{SD}$ : System-Dependability
$R_{pd}$ : Phase-Dependability, $0 < R_{pd} < 1$
$r_{cp}$ : Component-level Reliability within a Phase $'cp'$,
Where $0 < r_{cp} < 1$
$X_{cp}$ : Quantity of Components in a Phase $'cp'$
$P_{cp}$ : Price-Component within each Phase $'cp'$
$W_{cp}$ : Weight-Component within each Phase $'cp'$
$V_{cp}$ : Volume-Component within each Phase $'cp'$
$P_{gasp}$ : Maximum Permissible System-Price
$W_{gasw}$: Maximum Permissible System-Weight
$V_{gasv}$ : Maximum Permissible System-Volume
LMM : Lagrangean Multiplier Method
NRM : Newton-Raphson Method
IRRM : Integrated Redundant Reliability Model
$e_p, f_p, g_p, h_p, k_p, l_p$ are unchanging.

### 2.2 Computational Evaluation:

The system's effectiveness in relation to the given worth function

$$\text{Maximize } R_{SD} = 1 - \prod_{\beta=1}^{k}\left[1 - \prod_{p=1}^{n} R_{pd}\right]. \quad (1)$$

The worth coefficient of every unit in phase 'cp' is determined using the worth and efficiency relationship shown below.

$$r_{cp} = \cos^{-1}\left(\frac{P_{cp}}{e_p}\right)^{\frac{1}{f_p}}. \quad (2)$$

Therefore, $C_{cp} = e_p \cos[r_{cp}]^{f_p}$, (2a)
Similarly, $W_{cp} = g_p \cos[r_{cp}]^{h_p}$, (2b)
$V_{cp} = k_p \cos[r_{cp}]^{l_p}$. (2c)
Since price-components are linear in cp,
$\sum_{p=1}^{n} C_{cp} \cdot X_{cp} \le P_{gasp}$. (3a)
Similarly weight-components and volume-components are also linear in cp,
$\sum_{p=1}^{n} W_{cp} \cdot X_{cp} \le W_{gasw}$, (3b)
$\sum_{p=1}^{n} V_{cp} \cdot X_{cp} \le V_{gasv}$. (3c)
Substituting (2a), (2b) and (2c) in (3a), (3b) and (3c) respectively
$\sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} \cdot X_{cp} - P_{gasp} \le 0$, (4a)
$\sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} \cdot X_{cp} - W_{gasw} \le 0$, (4b)
$\sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} \cdot X_{cp} - V_{gasv} \le 0$. (4c)
The component equation is $X_{cp} = \frac{Ln[R_{pd}]}{Ln[r_{ep}]}$, (5)
Where $R_{pd} = \prod_{\beta=1}^{k}\left[1 - (1 - r_\beta)^{X_\beta}\right]$. (6)
Subject to the constraints

$$\sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} \cdot \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - P_{gasp} \le 0, \quad (7a)$$

$$\sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} \cdot \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - W_{gasw} \le 0, \quad (7b)$$

$$\sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} \cdot \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - V_{gasv} \le 0. \quad (7c)$$

Positivity restrictions cp $\ge 0$.
A Lagrangean function is defined as

$$L_F = R_{SD} + \vartheta_1 \left[\sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - P_{gasp}\right] + \vartheta_2 \left[\sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - W_{gasw}\right] + \vartheta_3 \left[\sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - V_{gasv}\right]. \quad (8)$$

The ideal point can be located and separated by using the Lagrangean function are $R_{SD}$ $r_{cp}$, $\vartheta_1$, $\vartheta_2$ and $\vartheta_3$.

$$\frac{\partial L_F}{\partial R_{SD}} = 1 + \vartheta_1 [\sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} . \frac{1}{R_{pd} Ln[r_{ep}]}] + \vartheta_2 \left[\sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} . \frac{1}{R_{pd} Ln[r_{ep}]}\right] + \vartheta_3 \left[\sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} . \frac{1}{R_{pd} Ln[r_{ep}]}\right], \quad (9)$$

$$\frac{\partial L_F}{\partial r_{cp}} = \vartheta_1 \left[\sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]}\right] \left[\frac{1}{Ln[1-r_{cp}]} - f_p . \tan r_{cp}\right] + \vartheta_2 \left[\sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]}\right] \left[\frac{1}{Ln[1-r_{cp}]} - h_p . \tan r_{cp}\right] + \vartheta_3 \left[\sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]}\right] \left[\frac{1}{Ln[1-r_{cp}]} - l_p . \tan r_{cp}\right], \quad (10)$$

$$\frac{\partial L_F}{\partial \vartheta_1} = \sum_{p=1}^{n} e_p \cos[r_{cp}]^{f_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - P_{gasp}, \quad (11)$$

$$\frac{\partial L_F}{\partial \vartheta_2} = \sum_{p=1}^{n} g_p \cos[r_{cp}]^{h_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - W_{gasw}, \quad (12)$$

$$\frac{\partial L_F}{\partial \vartheta_3} = \sum_{p=1}^{n} k_p \cos[r_{cp}]^{l_p} . \frac{Ln[R_{pd}]}{Ln[r_{ep}]} - V_{gasv}. \quad (13)$$

Where $\vartheta_1$, $\vartheta_2$ and $\vartheta_3$ are Lagrangean multipliers.

The Lagrangean technique is used to estimate the number of elements in each phase ($X_{cp}$),), the optimal component-reliability ($r_{cp}$), the stage-reliability ($R_{pd}$), and the structural reliability ($R_{SD}$). The optimal allocation of components in parallel-series systems is a critical aspect of reliability engineering. Various researchers have explored this area with a focus on exact algorithms, redundancy allocations, optimal assignment of interchangeable components, and heuristic approaches. This approach yields a concrete (numeric) solution with regard to price-component, weight-component and, volume-component.

## 2.3 Investigative Challenge

This study establishes assumptions regarding the link between price, weight, and volume components, among other things, and system dependability in the process of applying optimization algorithms to identify various parameters for a specific mechanical system. It's important to keep in mind that electronic systems could not be covered by this assumption. Therefore, the assessment of structural accuracy ($R_{SD}$), number of elements per stage ($X_{cp}$), stage-reliability ($R_{pd}$), and maximal component level reliability ($r_{cp}$) might be beneficial for any mechanical system.

An extensive literature study on optimal component assignment for parallel-series systems yields valuable new insights about exact algorithms, redundancy assignments, replaceable components, and heuristic techniques. These studies advance our understanding of reliability engineering in complicated systems. This study is specifically focused on assessing the structural accuracy of a specialized machine used in the assembly of single-phase AC synchronous generators. The schematic diagram of the AC synchronous generator is shown in Figure 1.

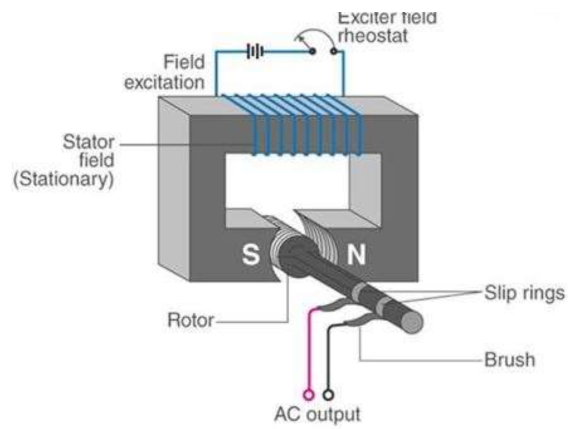


**Figure 1.** Basic AC Generator

A single-phase AC synchronous generator's cost, weight, and volume vary depending on construction, brand, and power capability. In general, single-phase generators designed for residential or small-scale applications tend to have a relatively lower cost, compact size, and lighter weight compared to their three-phase counterparts. The single-phase AC synchronous generator is priced at around $4440, encompassing the core cost of the machinery within the structure. Remarkably, the generator's weight stands at 2420 pounds, highlighting the substantial structural component it represents. Beyond the monetary value and weight, the generator occupies a volume of 390$cm^3$, emphasizing its compact size within the overall structure. It's important to note that these figures are illustrative and intended to accommodate potential adjustments to suit different contextual and environmental requirements, fostering inclusivity and adaptability across diverse scenarios.

### 2.4 Pre-fixed Constant Values for Components in the Case Study

Table 1 contains the information needed to calculate the constants for the case problem.

***Table1:*** *Component's-price, Component's-weight, and Component's-volume Pre-fixed Constant Values*

| Phase | Price & Its Reliability | | Weight & Its Reliability | | Volume & Its Reliability | |
|---|---|---|---|---|---|---|
| | $e_{cp}$ | $f_{cp}$ | $g_{cp}$ | $h_{cp}$ | $k_{cp}$ | $l_{cp}$ |
| 1 | 4210 | 0.91 | 2250 | 0.92 | 370 | 0.94 |
| 2 | 4300 | 0.94 | 2340 | 0.93 | 380 | 0.95 |
| 3 | 4440 | 0.95 | 2420 | 0.91 | 390 | 0.95 |

The effectiveness of individual factors, phases, and the quantity of factors in each stage, along with the structural effectiveness, is illustrated in the tables provided below.

#### 2.4.1 Implementing the Lagrangean Multiplier Technique with Exactness to Tackle Price-Component Constraints.

Table 2 describes the value-related efficiency design.

***Table2:*** *Price-Component Constraint Analysis by using Lagrangean Multiplier Method*

| Phase | $e_{cp}$ | $f_{cp}$ | $r_{cp}$ | $Log\ r_{cp}$ | $R_{pd}$ | $\text{Log } R_{pd}$ | $X_{cp}$ | $P_{cp}$ | $P_{cp} . X_{cp}$ |
|---|---|---|---|---|---|---|---|---|---|
| 01 | 4210 | 0.91 | 0.9057 | -0.0430 | 0.7804 | -0.1077 | 2.4295 | 3831.1 | 9307.6575 |
| 02 | 4300 | 0.94 | 0.9325 | -0.0304 | 0.7743 | -0.1111 | 3.5179 | 4042 | 14219.3518 |
| 03 | 4440 | 0.95 | 0.9489 | -0.0228 | 0.7628 | -0.1176 | 4.9319 | 4218 | 20802.7542 |
| Final Price-Component | | | | | | | | | 44329.7635 |

#### 2.4.2 Implementing the Lagrangean Multiplier Technique with Exactness to Tackle Weight-Component Constraints.

Table 3 describes the weight-related efficiency design.

***Table3:*** *Weight- Component Constraint Analysis by using Lagrangean Multiplier Method*

| Phase | $g_{cp}$ | $h_{cp}$ | $r_{cp}$ | $\text{Log } r_{cp}$ | $R_{pd}$ | $\text{Log } R_{pd}$ | $X_{cp}$ | $W_{cp}$ | $W_{cp} . X_{cp}$ |
|---|---|---|---|---|---|---|---|---|---|
| 01 | 2250 | 0.92 | 0.9057 | -0.0430 | 0.7804 | -0.1077 | 2.4295 | 2070 | 5029.0650 |
| 02 | 2340 | 0.93 | 0.9325 | -0.0304 | 0.7743 | -0.1111 | 3.5179 | 2176.2 | 7655.6540 |
| 03 | 2420 | 0.91 | 0.9489 | -0.0228 | 0.7628 | -0.1176 | 4.9319 | 2202.2 | 10861.0302 |
| Final Weight-Component | | | | | | | | | 23545.7492 |

#### 2.4.3 Implementing the Lagrangean Multiplier Technique with Exactness to Tackle Volume-Component Constraints.

Table 4 describes the volume-related efficiency design.

***Table4:*** *Volume-Component Constraint Analysis by using Lagrangean Multiplier Method*

| Phase | $k_{cp}$ | $l_{cp}$ | $r_{cp}$ | $\text{Log } r_{cp}$ | $R_{pd}$ | $\text{Log } R_{pd}$ | $X_{cp}$ | $V_{cp}$ | $V_{cp} . X_{cp}$ |
|---|---|---|---|---|---|---|---|---|---|
| 01 | 370 | 0.94 | 0.9057 | -0.0430 | 0.7804 | -0.1077 | 2.4295 | 347.8 | 844.9801 |
| 02 | 380 | 0.95 | 0.9325 | -0.0304 | 0.7743 | -0.1111 | 3.5179 | 361 | 1269.9619 |
| 03 | 390 | 0.95 | 0.9489 | -0.0228 | 0.7628 | -0.1176 | 4.9319 | 370.5 | 1827.2690 |
| Final Volume-Component | | | | | | | | | 3942.2110 |

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Structure Dependability($R_{SD}$), devoid of rounding-off, remains directly proportional to the individual components of price-component, weight-component, and volume-component with a constant of proportionality set at 0.9045.

## 3. ENHANCING EFFICIENCY VIA THE UTILIZATION OF THE LAGRANGEAN MULTIPLIER TECHNIQUE FOR OPTIMIZATION

The design efficiency [11] compiles the values of $'\alpha_j'$ integers, rounding each $'\alpha_j'$ value to the nearest whole number. Tables delineate the permissible results for, price-component, weight-component, and volume-component. The task at hand involves computing variances attributable to price-component, weight-component, and volume-component, and construction capacity, both before and after rounding off $'\alpha_j'$ to the nearest integer, to glean comprehensive insights.

### 3.1 Crafting Efficiency using the Lagrangean Multiplier Approach: Addressing Precision in Price, Weight, and Volume Components

***Table5:*** *The following table displays the efficiency design related to price-component, weight-component, and volume-component constraint analysis utilizing the Lagrangean multiplier approach with rounding-off.*

| Phase | $r_{cp}$ | $R_{pd}$ | $X_{cp}$ | $P_{cp}$ | $P_{cp} . X_{cp}$ | $X_{cp}$ | $W_{cp}$ | $W_{cp} . X_{cp}$ | $X_{cp}$ | $V_{cp}$ | $V_{cp} . X_{cp}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.9186 | 0.8115 | 3 | 4210 | 12630 | 3 | 2250 | 6750 | 3 | 370 | 1110 |
| 2 | 0.9452 | 0.8054 | 4 | 4300 | 17200 | 4 | 2340 | 9360 | 4 | 380 | 1520 |
| 3 | 0.9574 | 0.8073 | 5 | 4440 | 22200 | 5 | 2420 | 12100 | 5 | 390 | 1950 |
| Total price-Component, weight-Component, volume-Component | | | 52,030 | | | 28,210 | | | 4,580 | | |
| System Dependability ($R_{SD}$) | | | | | | | | | 0.9534 | | |

### 3.2 Comparison of LMM Approach Without-Rounding-off and With-Rounding-off relating to Price, Weight, and Volume Components

Under the framework of the Lagrangean multiplier method, we can determine the number of components, component reliability, stage reliability, and system reliability using a MATLAB program that is proportional to Price, Weight, and Volume. In this process, real-valued solutions were obtained, which are not suitable for the desired outcomes in the present IRRM study. Consequently, we applied the commonly used rounding-off method to obtain integer-valued solutions. The following results depict the variation between the LMM approach without rounding-off and the LMM approach with rounding-off.

3.2.1 Fluctuation in Price-Component
Total cost including rounding off – Total cost without rounding off / Total cost without rounding off = 17.37%
3.2.2 Fluctuation in Weight-Component
Total weight including rounding off – Total weight without rounding off / Total weight without rounding off = 19.81%
3.2.3 Fluctuation in Volume-Component
Total volume including rounding off – Total volume without rounding off / Total volume without rounding off = 16.18%
3.2.4 Fluctuation in Reliability
Efficiency including rounding off - Efficiency without rounding off / Efficiency without rounding off = 05.41%

## 4. NEWTON-RAPHSON APPROACH

Employing the Lagrangean technique, which possesses several drawbacks, including the requirement to specify the quantity of components needed at each stage ($'cp'$) in real numbers, can prove challenging to implement. The conventional practice of rounding down values may lead to alterations in price-component, weight-component, and volume-component, influencing system reliability and significantly impacting the dependability design of the model. Acknowledging this drawback, the writer suggests a different empirical strategy that uses the Newton-Raphson method to obtain an integer solution. This strategy makes use of the Lagrangean approach's solutions as parameters for the suggested Newton-Raphson procedure.

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### 4.1 Newton-Raphson Method

The Newton-Raphson method, also known as the Newton method, is an iterative numerical technique used to find approximate solutions of real-valued functions. It works especially well for locating a function's roots, also known as zeros. The technique is based on iteratively improving an initial guess using the tangent line approximation. This is how to apply the Newton-Raphson method step-by-step:

Step 1: Select a Ballpark Estimate: To begin, choose a preliminary estimate for the function's root. To guarantee convergence, this estimate needs to be somewhat near to the real root.

Step 2: Determine the Derivative and Function Value To determine the function value ($f(X)$) and its derivative ($f^1(X)$) at
that point, evaluate the function at the selected initial guess.
Step 3: Update the Approximation: To update the root approximation, use the tangent line approximation.

$$X_1 = X_0 - \frac{f(X_0)}{f^1(X_0)} \quad (14)$$

is the formula for updating the approximation. In this formula, $X_0$ represents the current approximation, while $X_1$ represents
the updated approximation.

$$\approx a + \frac{1}{2}\,\varepsilon_n^2\,\frac{f''(a)}{f'(a)}$$

(15g)

Therefore $t_{n+1} \approx a + \frac{1}{2}\,\varepsilon_n^2\,\frac{f''(a)}{f'(a)}$ (15h)

$$a - t_{n+1} = -\frac{1}{2}\,\varepsilon_n^2\,\frac{f''(a)}{f'(a)} \quad (15i)$$

i.e., $\varepsilon_{n+1} = -\frac{1}{2}\,\varepsilon_n^2\,\frac{f''(a)}{f'(a)}$ (15j)

Thus, the Newton's iteration formula is a second order process which means that the solution is the one of quadratic convergence.

### 4.3 Algorithm for the Newton's Method

Step 1: Choose a trail solution $t_\alpha$, $find\ f(t_0)\ and\ f'(t_0)$ (16a)
Step 2: Next approximation $x_i$ is obtained from
$t = t_0 - f(t_0)\,/\,f'(t_0)$ (16b)
Step 3: Follow the above procedure to find successive approximation $t_{r+1}$ using
the formula $t_{r+1} = t_r - \frac{f(t_r)}{f'(t_r)}$ $r = 1, 2, 3, \dots \dots .$ (16c)
Step 4: stop when $|t_{r+1} - t_r| < Q$, (16d)
Where Q is the prescribed accuracy.

## 5. RESULTS

The utilization of the Lagrangean multiplier method has provided a genuine numerical resolution for the examined mathematical models of Integrated Redundant Reliability Systems, meeting the requirement for a non-decimal solution. Employing a Newton-Raphson methodology, the researcher computed the updated phase reliability ($R_{pd}$), yielding values of 0.8424, 0.8243, and 0.8353 for the stage reliability ($R_{pd}$). The inquiry presents the results for the given mathematical function in tables 6, 7, and 8 sequentially using the Newton-Raphson technique, making it easier to draw important conclusions.

### 5.1 Exploring Constraints in Price, Weight, and Volume Components: A Comprehensive Analysis using the Newton's Raphson Method

Insights into Efficiency Design: Table 6 Details Price, Weight, and Volume-Component Relationships.

***Table6:*** *Constraint Analysis of Efficiency Design for Price, Weight, and Volume Components*

| Phase | $r_{cp}$ | $R_{pd}$ | $X_{cp}$ | $P_{cp}$ | $P_{cp} \cdot X_{cp}$ | $X_{cp}$ | $W_{cp}$ | $W_{cp} \cdot X_{cp}$ | $X_{cp}$ | $V_{cp}$ | $V_{cp} \cdot X_{cp}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 01 | 0.9266 | 0.8467 | 3 | 4010 | 12030 | 3 | 2175 | 6525 | 3 | 358 | 1074 |
| 02 | 0.9564 | 0.8384 | 4 | 4128 | 16512 | 3 | 2135 | 6405 | 3 | 362 | 1086 |
| 03 | 0.9781 | 0.8369 | 5 | 4255 | 21275 | 5 | 2248 | 11240 | 5 | 365 | 1825 |
| Total price-Component, weight-Component, volume-Component | | | 49817 | | | 24170 | | | 3985 | | |
| System Dependability ($R_{SD}$) | | | | | | 0.9534 | | | | | |

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### 5.2 Analyzing Optimization Strategies for Integrated Redundant Reliability Parallel-Series Systems: A Comparative Study of Lagrangean Multiplier Method (LMM) with Rounding-off and Newton's Raphson Approach for Price-Component

Analytical Contrast: Lagrangean Multiplier Method with Rounding-off vs. Newton's Raphson Method for Price-Related Efficiency Design – Insights from Table 7.

***Table7:*** *Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Newton's Raphson Method Approaches for Price-Component*

| | | LMM With Rounding Off | | | | Newton's Raphson Method | | | |
|---|---|---|---|---|---|---|---|---|---|
| Phase | $X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $P_{cp}$ | $P_{cp} . X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $P_{cp}$ | $P_{cp} . X_{cp}$ |
| 01 | *3* | 0.9186 | 0.8115 | 4210 | *12630* | 0.9266 | 0.8467 | 4010 | *12030* |
| 02 | *4* | 0.9452 | 0.8054 | 4300 | *17200* | 0.9564 | 0.8384 | 4128 | *16512* |
| 03 | *5* | 0.9574 | 0.8073 | 4440 | *22200* | 0.9781 | 0.8369 | 4255 | *21275* |
| Final- Price-Component | | 52030 | | | | 49817 | | | |
| System Dependability ($R_{SD}$) | | Applying the LMM Methodology | | | 0.9534 | Applying the NRM Methodology | | | 0.9633 |

### 5.3 Analyzing Optimization Strategies for Integrated Redundant Reliability Parallel-Series Systems: A Comparative Study of Lagrangean Multiplier Method (LMM) with Rounding-off and Newton's Raphson Approach for Weight-Component

Analytical Contrast: Lagrangean Multiplier Method with Rounding-off vs. Newton's Raphson Method for Weight-Related Efficiency Design – Insights from Table 8.

***Table8:*** *Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Newton's Raphson Method Approaches for Weight-Component*

| | | LMM With Rounding Off | | | | Newton's Raphson Method | | | |
|---|---|---|---|---|---|---|---|---|---|
| Phase | $X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $W_{cp}$ | $W_{cp} . X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $W_{cp}$ | $W_{cp} . X_{cp}$ |
| 01 | 3 | 0.9186 | 0.8115 | 2250 | *6750* | 0.9266 | 0.8467 | 2175 | *6525* |
| 02 | 3 | 0.9452 | 0.8054 | 2340 | *7020* | 0.9564 | 0.8384 | 2135 | *6405* |
| 03 | 4 | 0.9574 | 0.8073 | 2420 | *9680* | 0.9781 | 0.8369 | 2248 | *8992* |
| Final-Weight-Component | | 234505.4.1 Fluctuation in Price-Component under NRM Frame Work = 04.25% | | | | 21922 | | | |
| System Dependability ($R_{SD}$) | | Applying the LMM Methodology | | | 0.9534 | Applying the NRM Methodology | | | 0.9633 |

### 5.4 Analyzing Optimization Strategies for Integrated Redundant Reliability Parallel-Series Systems: A Comparative Study of Lagrangean Multiplier Method (LMM) with Rounding-off and Newton's Raphson Approach for Volume- Component

Analytical Contrast: Lagrangean Multiplier Method with Rounding-off vs. Newton's Raphson Method for Volume-Related Efficiency Design – Insights from Table 9.

***Table9:*** *Correlating Results: Comparative Analysis of Lagrangean Multiplier Method with Rounding-off and Newton's Raphson Method Approaches for Volume-Component*

| | | LMM With Rounding Off | | | | Newton's Raphson Method | | | |
|---|---|---|---|---|---|---|---|---|---|
| Phase | $X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $W_{cp}$ | $W_{cp} . X_{cp}$ | $r_{cp}$ | $R_{pd}$ | $W_{cp}$ | $W_{cp}.X_{cp}$ |
| 01 | 3 | 0.9186 | 0.8115 | 370 | *1110* | 0.9266 | 0.8467 | 358 | *1074* |
| 02 | 4 | 0.9452 | 0.8054 | 380 | *1520* | 0.9564 | 0.8384 | 362 | *1086* |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 03 | 5 | 0.9574 | 0.8073 | 390 | *1950* | 0.9781 | 0.8369 | 365 | *1825* |
| Final-Volume-Component | | 4580 | | | | 3620 | | | |
| System Dependability ($R_{SD}$) | | Applying the LMM Methodology | | | 0.9534 | Applying the NRM Methodology | | | 0.9633 |

5.4.1 Fluctuation in Price-Component under NRM Frame Work = 04.25%

5.4.2 Fluctuation in Weight-Component under NRM Frame Work = 06.52%

5.4.3 Fluctuation in Volume-Component under NRM Frame Work = 04.99%

5.4.4 Fluctuation in System Dependability under NRM Frame Work = 01.04%

## 6. DISCUSSION

This study introduces an original reliability framework tailored for a parallel-series configuration system with multiple efficiency criteria. Dealing with information that is represented by real numbers, the Lagrangean multiplier approach is utilized to ascertain the values of components ($X_{cp}$), component reliability ($r_{cp}$), stage reliability ($R_{pd}$), and system reliability ($R_{SD}$). The resulting component efficiencies ($r_{cp}$) are 0.9186, 0.9452, and 0.9574, stage reliabilities ($R_{pd}$) are 0.8115, 0.8054, and 0.8073, and structure dependability ($R_{SD}$) is 0.9534.

For practical application, the Newton's Raphson method is employed to acquire integer solutions, resulting in component reliabilities ($r_{cp}$) of 0.9266, 0.9564, and 0.9781, stage reliabilities ($R_{pd}$) of 0.8467, 0.8384, and 0.8369, and system reliability ($R_{SD}$) of 0.9633. These integer solutions are derived from inputs obtained through the Lagrangean method, ensuring the model's practical relevance.
In the context of our findings on Integrated Redundant Reliability (IRR) models, an examination of the Newton-Raphson method in comparison with weight and volume components reveals a noteworthy influence of cost components. Unlike weight and volume components, cost components exhibit fluctuations during the application of the Newton-Raphson method. Despite the absence of fluctuations in weight and volume components using the same approach, our findings suggest that targeted reduction of cost components has the potential to concurrently enhance the stage and system reliability within the proposed IRR model. This observation underscores the strategic significance of cost optimization in the pursuit of improved reliability.

The analysis discloses subtle variations in price-component, weight-component, and volume-component, though minor. Nevertheless, when juxtaposed with stage reliability, these fluctuations positively impact overall system reliability. The developed Integrated Reliability Model (IRM) proves highly valuable, particularly in practical scenarios where reliability engineers need to incorporate redundancy within a parallel-series configuration, especially when the system's intrinsic value is relatively low.

In order to maximize system dependability, the authors suggest that future research look into a novel methodology that places limitations on the minimum and maximum reliability values of components. Drawing on existing heuristic processes, the goal is to formulate analogous Integrated Reliability Models (IRMs) featuring redundancy, thereby expanding the applicability and versatility of such models.

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## REFERENCES

[1] Mishra, K.B. (1972). Reliability optimization of a series-parallel system. *IEEE Transactions on Reliability*, 21(4), 230-238. DOI:10.1109/TR.1972.5216000.
[2] Mishra, K.B. (1971). A method of solving redundancy optimization problems. *IEEE Transactions on Reliability*, 20(3), 117-20. DOI:10.1109/TR.1971.5216110
[3] Agarwal, K.K., & Gupta, J.S. (1975). On minimizing the expense of reliable systems. *IEEE Transactions on Reliability*, 24(3), 205-215. DOI:10.1109/TR.1975.5215153.
[4] Agarwal, K.K., Mishra, K.B., & Gupta, J.S. (1975). Reliability evaluation a comparative study of different

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techniques. *Micro Electronics Reliability*, 14(1), 49-56. https://doi.org/10.1016/0026-2714(75)90461-8.

[5] Ei-Neweihi, E.F., Proschan & Sethuraman, J. (1986). Optimal allocation of components in parallel – series and series – parallel system. *Journal of Applied Probability*, 23(01),770-777. https://doi.org/10.2307/3214014.

[6] Prasad, V.R., Aneja, Y.P., & Nair, K.P.K. (1991). A heuristic approach to optimal assignment of components to a parallel-series network. *IEEE Transaction of Reliability*, 40(02), 555-558. DOI: 10.1109/24.106776.

[7] Baxter, P., L.A., & HARCHE, F. (1992). On the optimal assembly of series parallel systems. *Operations Research Letters*, 11(1), 153-157. https://doi.org/10.1016/0167-6377(92)90079-I

[8] Fan, L.T., & Wang, T. (1997). Optimization of system reliability. *IEEE Transactions on Reliability*, 16(2), 81-86. DOI:10.1109/TR.1967.5217465.

[9] Prasad, V.R., & Ragawachari. (1998). Optimal allocation of interchangeable components to series–parallel reliability system. *IEEE Transaction of Reliability*, 47(03), 255-260. DOI: 10.1109/24.740496.

[10] Kuo, W., & Prasad, V.R. (2000). An annotated overview of system-reliability optimization. *IEEE Transactions on Reliability*, 49(2), 176-187. DOI:10.1109/24.877336.

[11] Ho-Gyun Kim and Chang-Ok Bae (2006). Reliability-redundancy optimization using simulated annealing algorithms. *Journal of Quality in Maintenance Engineerin,* 12(4), 354-363. DOI 10.1108/13552510610705928

[12] Sridhar, A., Sasikala, P., Kumar, S.P., & Sarma, Y.V.S. (2021). *Chapter 9* - Design and evaluation of coherent redundant system reliability. *Systems Reliability Engineering, edited by Amit Kumar and Mangey Ram, De Gruyte*r, pp. 137-152. https://doi.org/10.1515/9783110617375.

[13] Akiri, S., Sasikala, P., Subbara, P. K., & Yadavalli, V. S. S. (2022). Design and evaluation of parallel-series IRM system. *In System Assurances*, pp. 189-208.

.[14] Srinivasa Rao V., Sridhar Akiri, Kesava Rao Y. (2022). Design and Implementation of An Integrated Redundant Reliability System for k-out-of-n Configuration. *Journal of Neuroquantology*, 20(19), 998-1009. https://doi.org/0.48047/nq.2022.20.19.NQ99091

[15] Srinivasa Rao, V., Sridhar Akiri, Pavan Kumar, S. (2022). k-out-of-n Systems Growth Study Focusing on Redundant Reliability Systems by using Heuristic Programming Approach. *Mathematical Statistician and Engineering Applications*, 71(4), 5602-5613. https://doi.org/10.17762/msea.v71i4.1153.

[16] Srinivasa Rao Velampudi, Sridhar Akiri, Pavan Kumar Subbara & Yadavalli V S S., (2023), A Comprehensive Case Study on Integrated Redundant Reliability Model using k out of n Configuration, *Journal of Reliability Theory & Applications*. 18(1[72]), 110-119. https://doi.org/10.24412/1932-2321-2023-172-110-119.

[17] Achintya Roy & Nitin Gupta (2024), A study on utilization of two cold standby components to increase reliability of a coherent system, *Communications in Statistics - Theory and Methods*. 53(06), 668-683. https://doi.org/10.1080/03610926.2024.2309981

[18] Gregory Levitin, Liudong Xing, Hanoch Benhaim & Yiwu Dai., (2019), Optimal structure of series system with 1-out-of-n warm standby subsystems performing operation and rescue functions. *Journal of Reliability Engineering & System Safety*. 188(4), 523-531. https://doi.org/10.1016/j.ress.2019.04.005.