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# COMPARATIVE ANALYSIS ON THE PERFORMANCE OF NHPP-BASED SOFTWARE RELIABILITY MODEL FOLLOWING EXPONENTIAL LIFE DISTRIBUTION

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**ABSTRACT**

In this study, the exponential-type life distribution, which is known to be suitable for reliability analysis of failure occurrence phenomena, was applied to the NHPP-based software reliability model, and then the performance of the applied model was newly studied. For this purpose, the failure time data requested by the developer was used. In conclusion, first, as a result of evaluating MSE and $R^2$, which are the selection criteria for efficient models to determine the suitability of the proposed model, the proposed models showed a performance of over 85% and were all efficient. Second, in the performance analysis using m(t) and λ(t), which are attribute functions that have a significant impact on the performance of the reliability model, the Inverse-exponential model showed excellent performance. Third, as a result of testing future reliability, the reliability of the Inverse-exponential and Rayleigh models showed a high and stable trend over time. Therefore, as a result of evaluating performance attribute data (MSE and $R^2$, m(t), λ(t), $\hat{R}(\tau)$), it was concluded that the Inverse-exponential model had the best performance. Thus, this study can present basic design data along with algorithmic solution techniques that can analyze and predict performance attribute data needed by developers during the early software development process.

**Keywords:** *Exponential-basic, Exponential-type, Exponential-power, Inverse-exponential, Rayleigh, Software Reliability.*

## 1. INTRODUCTION

The core of the 4th industrial revolution era will be artificial intelligence technology. Artificial intelligence technology is already being used in combination with software not only in industrial settings but also throughout our lives. Therefore, when the era of active artificial intelligence arrives, the processing we have been doing so far will be further converged based on software technology, and unnecessary parts will be eliminated. For this reason, software technology will play the most critical role not only in the early stages of artificial intelligence but also in the future artificial intelligence era. Therefore, related researchers regard reliability issues as the core topic in the step of developing highly reliable software. To solve this problem, developers are paying attention to reliability research applying the non-homogeneous Poisson Process (NHPP) [1]. Regarding the software reliability issue in this study, Park [2] analyzed reliability performance using the NHPP-type reliability model and presented attribute data needed by developers using the non-exponential family distribution. Xiao and Dohi [3] newly presented the properties of the NHPP-based reliability model applying the Weibull distribution through goodness-of-fit evaluation and parameter prediction. Kapur, Singh, Shatnawi, and Gupta [4] proposed a new NHPP-type discrete reliability model and demonstrated improved performance in error and defect removal. Therefore, Pham and Zhang [5] proposed a model suitable for quantitatively predicting software reliability, and Okamura and Dohi [6] proposed a new-type NHPP model and applied it to real data to solve the model's efficiency problem. In particular, Bae [7] used the Weibull distribution to convert the NHPP-based cost model to be applicable to the software system and also analyzed the attribute relationship between cost and release time. Based on existing research data, Yang [8] presented a new software reliability model by combining the NHPP model with log-type distributions, and also solved attribute problems related to reliability. Kim [9] proposed a technique to solve the suitability problem of the reliability model by the property of Exponential-exponential distribution. Therefore, Yang [10] presented attribute data required for performance evaluation of NHPP-based reliability model and implemented an algorithm to solve this problem.

Thus, in this work, the Exponential-type distribution model was applied to the NHPP-based software reliability model. Based on this, the performance of the proposed model was studied and explored using the newly proposed algorithm. Also, we would like to present the most efficient model among the exponential-family distribution models proposed based on the presented data.

## 2. RELATED RESEARCH

### 2.1.1 NHPP Model

In an NHPP-based system, if N(t) is the number of failures per unit when the failure time intervals are different, N(t) follows a Poisson distribution and satisfies the non-homogeneity of different failure time intervals. Therefore, the NHPP model can be said to be very useful for modeling software systems. Also, the NHPP-based software reliability model can be said to be a model that can predict future failures.

In the NHPP-based model, N(t) represents the cumulative number of defects, and m(t) is the average value of failures occurring at a given time t.

Therefore, m(t) becomes an attribute function that can predict the probability of failure that will occur in the future. Thus, the NHPP software reliability model is as follows.

$$P\{N(t)=n\}=\frac{[m(t)]^n\cdot e^{-m(t)}}{n!} \tag{1}$$

Note that $n=0,1,2,\cdots\infty$.

Where m(t) refers to the mean value function that has the property of predicting the true value and can be defined as Equation (2). Therefore, the intensity function λ(t), which has properties representing the instantaneous failure rate, can be obtained by differentiating Equation (2).

$$m(t)=\int_0^t \lambda(s)ds \tag{2}$$

$$\lambda(t)=\frac{dm(t)}{d(t)} \tag{3}$$

### 2.1.2 NHPP Software Reliability Model

In this study, we attempt to solve attribute problems regarding the reliability performance of the proposed NHPP-based model applying software failure time data. This study reflects the failure phenomenon of generally developed software and aims to study it based on finite failure, in which no further failures occur after the failure is repaired. Therefore, reflecting realistic software failure situations, this study will develop a research topic based on the finite failure NHPP reliability model.

Therefore, if the remaining failure rate that can be discovered by time t using the relationships of Equations (2) and (3) is θ, then the attribute function representing reliability performance can be written as follows.

$$\mathrm{m}(\mathrm{t}|\theta,\mathrm{b})=\theta F(\mathrm{t}) \tag{4}$$

$$\lambda(\mathrm{t}|\theta,\mathrm{b})=\theta F(\mathrm{t})'=\theta f(t) \tag{5}$$

Note that F(t) is the cumulative distribution function.

Accordingly, m(t) represents a property that can predict the true value, and λ(t) represents a property that represents the intensity at which a failure may occur. Therefore, the likelihood function of the finite failure NHPP model is as follows.

$$L_{NHPP}(\Theta|\underline{x})=\left(\prod_{i=1}^{n}\lambda(x_i)\right)exp[-m(x_n)] \tag{6}$$

Note that $\underline{x}=(x_1,x_2,x_3\cdots x_n)$

### 2.2 NHPP Exponential-basic Model

The Exponential-basic model, widely known as the most basic NHPP model, has the basic concept that the number of failures discovered per unit time is proportional to the number of failures remaining at that time.

Also, the Exponential-basic model has exponential-type life distribution characteristics, so it is also called the Goel-Okumoto basic model, which is a basic exponential-type distribution model.

In the NHPP finite failure situation presented in this study, if the expected value of the defect causing the failure is expressed as θ and the defect search rate is b, b was considered a fixed constant. Therefore, since the failure rate per software fault in the Exponential-basic model can be considered a constant value with a certain form, the performance attribute function can be written as follows [11].

$$m(t|\theta, b) = \theta(1 - e^{-bt}) \tag{7}$$

$$\lambda(t|\theta, b) = \theta b e^{-bt} \tag{8}$$

Therefore, the log-likelihood function of this model calculated by applying Equation (6) is as follows.

$$lnL_{NHPP}(\Theta|\underline{x}) = nln\theta + nlnb - b\sum_{k=1}^{n} x_k - \theta(1 - e^{-bx_n}) \tag{9}$$

That is, the parameter estimator ( $\hat{\theta}_{MLE}$, $\hat{b}_{MLE}$) of the Exponential-basic model to be obtained in this work can be calculated by applying Maximum Likelihood Estimation (MLE) to Equation (9) and then using the bisection method. Therefore, Equations (10) and (11) show the final equations for calculating the parameters.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \tag{10}$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta} x_n e^{-\hat{b}x_n} = 0 \tag{11}$$

**2.3 NHPP Exponential-power Model**

The Exponential-power distribution is a distribution consisting of shape parameters and scale parameters that can represent various phenomena, and is used in the field of software reliability. Also, the function F(t) of this model is as follows.

$$\mathrm{F(t)} = \left(1 - e^{1-e^{\delta t^k}}\right) \tag{12}$$

Thus, if applying the Equation (12), when the shape parameter k = 2, the probability density function has a symmetrical form, so it is not suitable as a life distribution. But, in the case of the shape parameter k = 1, it can be seen that the probability density function is suitable as a life distribution because it shows a decreasing form (reliability growth) as the failure time passes.

Therefore, the case of shape parameter k=1 is applied and analyzed.

That is, by applying Equations (4) and (5), the attribute function representing reliability performance can be solved as follows [12].

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = \theta\left(e^{1-e^{\delta t}}\right) e^{\delta t} \tag{13}$$

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta\left(1 - e^{1-e^{\delta t}}\right) \tag{14}$$

Therefore, the log-likelihood function of this model calculated by applying Equation (6) is as follows.

$$ln\, L_{NHPP}(\Theta|\underline{x}) = n\ln\theta + \sum_{i=1}^{n}(1 - e^{\delta x_i}) + \delta\sum_{i=1}^{n} x_i - \theta\left(1 - e^{1-e^{\delta x_n}}\right) \tag{15}$$

Accordingly, the parameter estimator ($\hat{\theta}_{MLE}$, $\hat{\delta}_{MLE}$ ) of the Exponential-power model to be obtained in this study can be calculated by applying MLE to Equation (15) and then using the bisection method.

Thus, Equations (16) and (17) show the final equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} - \left(1 - e^{1-e^{\delta x_n}}\right) = 0 \tag{16}$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \delta} = -\sum_{i=1}^{n} x_i\, e^{\delta x_i} + \sum_{i=1}^{n} x_i - \theta x_n e^{\delta x_n}\left(e^{1-e^{\delta x_n}}\right) = 0 \tag{17}$$

**2.4 NHPP Inverse-exponential Model**

Unlike the exponential-type distribution, which has a constant risk rate function, the Inverse-exponential distribution has a bathtub-shaped risk rate function, so it is a useful distribution for the analysis of reliability life data with the characteristic that the risk rate varies with time.

In particular, the Inverse-exponential distribution is known as a distribution suitable for load-intensity reliability, which represents the probability that the system will operate normally when stress is applied to the system stochastically.

Therefore, the Inverse-exponential distribution plays a very important role in measuring the reliability of a system. Accordingly, the function F(t) of this model is as follows.

$$\mathrm{F(t)} = e^{-(bt)^{-1}} \tag{18}$$

Thus, by applying Equations (4) and (5), the attribute function can be obtained as follows [13].

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta e^{-(bt)^{-1}} \tag{19}$$

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = \theta b^{-1} t^{-2} e^{-(bt)^{-1}} \tag{20}$$

Therefore, the log-likelihood function of this model derived by applying Equation (6) is as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta - nlnb + 2\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}(bx_i)^{-1} - \hat{\theta} e^{-(bx_n)^{-1}} = 0 \tag{21}$$

That is, the parameter estimator ($\hat{\theta}_{MLE}$, $\hat{b}_{MLE}$) of the Inverse-exponential model to be obtained in this work can be calculated by applying MLE to Equation (21) and then using the bisection method.

Thus, Equations (22) and (23) show the final equations for calculating the parameters.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - e^{-(\hat{b}x_n)^{-1}} = 0 \tag{22}$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = -\frac{n}{\hat{b}} + \frac{1}{\hat{b}^2}\sum_{i=1}^{n}\frac{1}{x_i} - \theta\frac{1}{b^2 x_n} e^{-(\hat{b}x_n)^{-1}} = 0 \tag{23}$$

**2.5 NHPP Rayleigh Model**

The Rayleigh distribution was originally used as a distribution to analyze distances in electromagnetic and spatial Poisson processes, but has recently been widely used as a lifetime test to analyze reliability.

In particular, the Rayleigh distribution, which has exponential-type distribution characteristics, can be said to be a special form in which the shape parameter (α) is 2 in the Weibull distribution.

Here, if replaced with $\frac{1}{2\beta^2} = b$ and simplified, it is as follows.

$$F(\mathrm{t}) = \left(1 - e^{-\frac{t^\alpha}{2\beta^2}}\right) = \left(1 - e^{-bt^\alpha}\right) \tag{24}$$

$$\mathrm{f(t)} = \left(\frac{t^{\alpha-1}}{\beta^2} e^{-\frac{t^\alpha}{2\beta^2}}\right) = \left(2\mathrm{b}t^{a-1} e^{-bt^\alpha}\right) \tag{25}$$

Note that $\beta > 0,\ t \in [0,\ \infty]$.

When the shape prameter (α) in the Weibull distribution is 2, the Rayleigh distribution is established. By applying Equations (4) and (5), the attribute function representing reliability performance can be obtained as follows [14].

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta(1 - e^{-bt^2}) \tag{26}$$

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = 2\theta \mathrm{bt} e^{-bt^2} \tag{27}$$

Therefore, the log-likelihood function of this model is as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln2 + nln\theta + nlnb + \sum_{i=1}^{n} lnx_i - b\sum_{i=1}^{n} {x_i}^2 - \theta\left(1 - e^{-bx_n^2}\right) \tag{28}$$

Accordingly, the parameter estimator ($\hat{\theta}_{MLE}$, $\hat{b}_{MLE}$) of the Rayleigh model to be obtained in this study can be calculated by applying MLE to Equation (28) and then using the bisection method. Therefore, Equations (29) and (30) show the final calculation equations for calculating the parameters.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + exp(-\hat{b}x_n^2) = 0 \quad (29)$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_i^2 \quad (30)$$

$$-\hat{\theta}x_n^2\, exp(-\hat{b}x_n^2) = 0$$

## 3. RELIABILITY PERFORMANCE ANALYSIS

In this study, the following optimization algorithm was proposed to analyze the reliability performance of the applied model.

Accordingly, based on the proposed algorithm, the solution method was presented and developed step by step (step. 1 to step. 5).

**Step 1:** Verification of reliability of applied data (software failure time).

**Step 2:** Calculation the maximum likelihood estimator $(\hat{\theta}, \hat{b})$ of the proposed model.

**Step 3:** Properties (MSE, $R^2$) analysis for efficient model selection.

**Step 4:** Analysis of performance attribute functions (m(t), λ(t), $\hat{R}(\tau)$).

**Step 5:** Presentation of performance evaluation data needed by software developer.

Also, to evaluate the reliability performance of the proposed model, the software failure time data required by the developer during the development process was used as shown in Table 1 [15].

This data is a collection of a total of 30 failures that occurred irregularly during the normal operating time of the software system (187.35 hours).

*Table 1: Software Failure Time Data.*

| Failure number | Failure time (hours) | Failure time (hours)$\times 10^{-1}$ |
|---|---|---|
| 1 | 4.79 | 0.479 |
| 2 | 7.45 | 0.745 |
| 3 | 10.22 | 1.022 |
| 4 | 15.76 | 1.576 |
| 5 | 26.10 | 2.610 |
| 6 | 35.59 | 3.559 |
| 7 | 42.52 | 4.252 |
| 8 | 48.49 | 4.849 |
| 9 | 49.66 | 4.966 |
| 10 | 51.36 | 5.136 |
| 11 | 52.53 | 5.253 |
| 12 | 65.27 | 6.527 |
| 13 | 69.96 | 6.996 |
| 14 | 81.70 | 8.170 |
| 15 | 88.63 | 8.863 |
| 16 | 107.71 | 10.771 |
| 17 | 109.06 | 10.906 |
| 18 | 111.83 | 11.183 |
| 19 | 117.79 | 11.779 |
| 20 | 125.36 | 12.536 |
| 21 | 129.73 | 12.973 |
| 22 | 152.03 | 15.203 |
| 23 | 156.40 | 15.640 |
| 24 | 159.80 | 15.980 |
| 25 | 163.85 | 16.385 |
| 26 | 169.60 | 16.960 |
| 27 | 172.37 | 17.237 |
| 28 | 176.00 | 17.600 |
| 29 | 181.22 | 18.122 |
| 30 | 187.35 | 18.735 |

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**3.1. Step 1: Verification of Reliability of Applied Data (Software Failure Time).**

In this work, the applicability of the cited failure time data was verified applying the Laplace trend test method. In general, if the analyzed data is distributed between '-2 and 2', it is reliable [16].

Figure 1 shows the analysis result of a Laplace trend test for the quoted failure time data. Therefore, it can be said that the data presented in Table 1 is distributed between 0 and 2 and can be applied for this work.

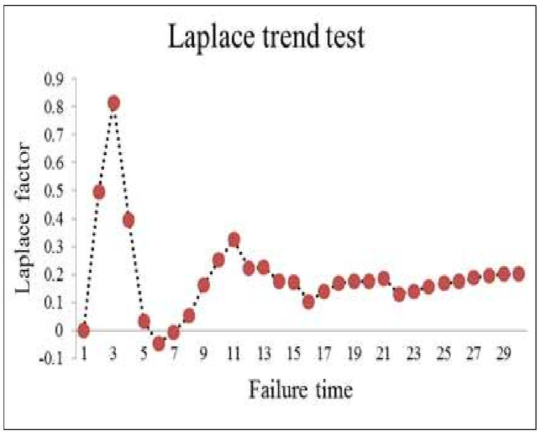


*Figure 1: Results of Laplace Trend Test.*

**3.2. Step 2: Calculation the Maximum Likelihood Estimator $(\hat{\theta}, \hat{b})$ of the Proposed Model.**

In this work, MLE method was used to solve the parameter estimators $(\hat{\theta}, \hat{b})$. Thus, Table 2 shows the parameter (maximum likelihood estimator) values obtained by applying the MLE.

*Table 2: Parameter values using MLE.*

| Type | NHPP model | MLE $\hat{\theta}$ | MLE $\hat{b}(\hat{\delta})$ |
|---|---|---|---|
| Basic | Exponential-basic | 32.9261 | 0.12970 |
| Exponential life distribution | Exponential-power | 54.9038 | 0.03109 |
| | Inverse-exponential | 41.2881 | 0.16920 |
| | Rayleigh | 30.0412 | 0.01880 |

**3.3. Step 3: Properties (MSE, $R^2$) Analysis for Efficient Model Selection.**

In this work, MSE and $R^2$ method were applied as verification data to confirm the efficiency of the model [17].

**3.3.1. Mean Square Error (MSE)**

MSE is a tool that measures the difference between actual observed values and predicted values. The smaller this value is, the more accurate the prediction is, making it an efficient model. Therefore, it is defined as Equation (31).

$$MSE = \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \quad (31)$$

Note that $\hat{m}(x_i)$ represents the cumulative number of failures.

Figure 2 is the result of analyzing the model properties using MSE, and this study intends to use this value as standard data to determine the suitability of the model along with the efficiency of the cited model.

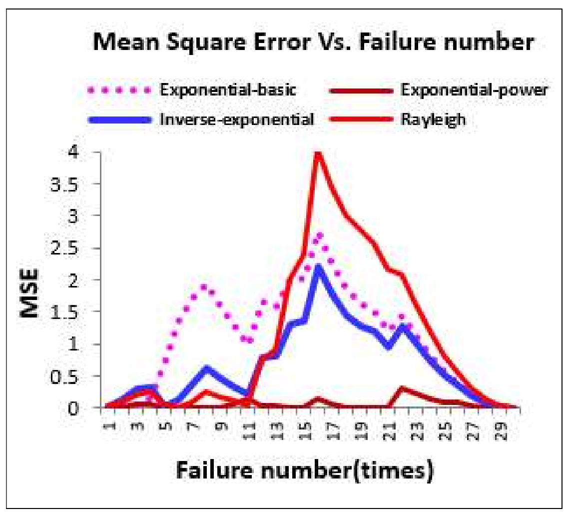


*Figure 2: Result of MSE Analysis.*

**3.3.2. Coefficient of Determination $(R^2)$**

$R^2$ is a tool that represents the difference between observed and predicted values. The larger this value, the greater the explanatory power of the true value, making it an efficient model. Therefore, it is defined as Equation (32).

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$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (32)$$

In this study, we intend to use this value as standard data to determine the suitability of the model along with the efficiency of the proposed model.

Table 3 shows the results of calculating MSE and $R^2$ values, which are applied as reference values in the process of generally selecting an efficient model.

*Table 3: Analysis of Model Efficiency.*

| Type | NHPP model | MSE | $R^2$ |
|---|---|---|---|
| Basic | Exponential-basic | 32.9379 | 0.8956 |
| Exponential family distribution | Exponential-power | 1.6128 | 0.9948 |
| | Inverse-exponential | 20.2035 | 0.9359 |
| | Rayleigh | 32.1798 | 0.8980 |

### 3.4. Step 4: Analysis of Performance Attribute Functions (m(t), $\lambda(t)$, $\hat{R}(\tau)$).

#### 3.4.1 Mean Value Function ($m(t)$)

Figure 3 shows the trend of performance attributes analyzed for the reliability of the proposed model using the attribute function m(t), which has a significant impact on the performance of the software reliabilty model. Additionally, by referring to these attribute data, the predictive ability to estimate the true value can be analyzed.

Therefore, when analyzing the attribute function m(t), the Exponential-power model, which showed a tendency to predict the true value with the smallest error, can be said to be efficient.

Table 4 shows data analyzed by detailed comparison for each number of failures by applying failure times that occurred for a total of 187.35 hours to the attribute functions (m(t), $\lambda(t)$).

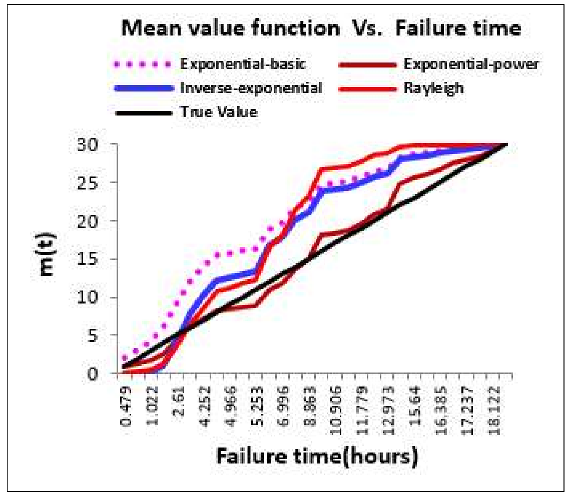


*Figure 3: Result of m(t) Analysis.*

#### 3.4.2 Intensity Function ($\lambda(t)$)

The property of the intensity function is that the failure rate generally shows an increasing trend before the defect is repaired, but as time passes, the failure rate eventually decreases because the occurring failure is repaired.

Figure 4 shows the trend of performance properties for the intensity function representing the intensity of occurrence of instantaneous failures. The properties of the Inverse-exponential and Exponential-power models showed that the failure rate increased in the early stages before the failure was repaired, and then gradually decreased over time. Thus, these models can be said to be excellent in terms of reliability property [18].

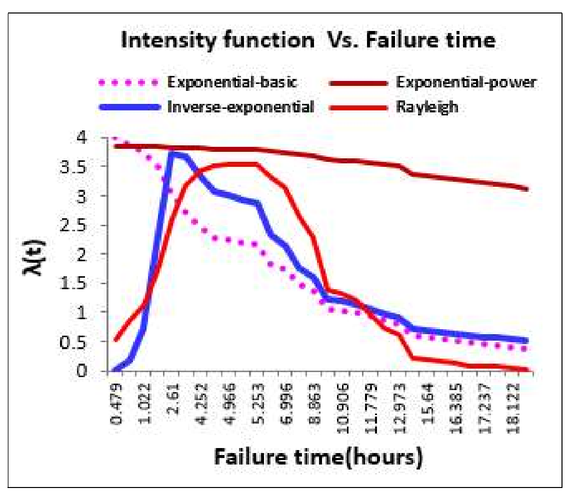


*Figure 4: Result of $\lambda(t)$ Analysis.*

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*Table 4: Detailed Analysis Data of Reliability Performance.*

| Failure Number | Reliability Performance | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | m(t) | | | | λ(t) | | | |
| | Exponential -basic | Exponential -power | Inverse-exponential | Rayleigh | Exponential -basic | Exponential -power | Inverse-exponential | Rayleigh |
| 1 | 1.9833 | 0.8176 | 0.0001 | 0.1293 | 4.0132 | 3.8428 | 0.0046 | 0.5387 |
| 2 | 3.0326 | 1.2715 | 0.0148 | 0.3118 | 3.8771 | 3.8422 | 0.1576 | 0.8327 |
| 3 | 4.0875 | 1.7442 | 0.1271 | 0.5841 | 3.7403 | 3.8413 | 0.7194 | 1.1319 |
| 4 | 6.08703 | 2.6890 | 0.9709 | 1.3705 | 3.4810 | 3.8385 | 2.3102 | 1.6989 |
| 5 | 9.45549 | 4.4501 | 4.2893 | 3.6111 | 3.0441 | 3.8302 | 3.7214 | 2.5937 |
| 6 | 12.1736 | 6.0623 | 7.8455 | 6.3657 | 2.6915 | 3.8189 | 3.6607 | 3.1682 |
| 7 | 13.9575 | 7.2361 | 10.2841 | 8.6565 | 2.4602 | 3.8083 | 3.3618 | 3.4188 |
| 8 | 15.3708 | 8.2445 | 12.2036 | 10.7330 | 2.2769 | 3.7975 | 3.0674 | 3.5203 |
| 9 | 15.6352 | 8.4418 | 12.5591 | 11.1453 | 2.2426 | 3.7953 | 3.0098 | 3.5282 |
| 10 | 16.0123 | 8.7283 | 13.0637 | 11.7457 | 2.1937 | 3.7919 | 2.9269 | 3.5331 |
| 11 | 16.2670 | 8.9252 | 13.4029 | 12.1590 | 2.1606 | 3.7894 | 2.8706 | 3.5319 |
| 12 | 18.8043 | 11.0613 | 16.6945 | 16.5551 | 1.8315 | 3.7594 | 2.3160 | 3.3096 |
| 13 | 19.6377 | 11.8431 | 17.7392 | 18.0709 | 1.7234 | 3.7466 | 2.1420 | 3.1487 |
| 14 | 21.5146 | 13.7875 | 20.0288 | 21.4761 | 1.4800 | 3.7104 | 1.7734 | 2.6311 |
| 15 | 22.4955 | 14.9259 | 21.1944 | 23.1807 | 1.3528 | 3.6862 | 1.5946 | 2.2862 |
| 16 | 24.7822 | 18.0181 | 23.8518 | 26.6489 | 1.0562 | 3.6090 | 1.2150 | 1.3738 |
| 17 | 24.9235 | 18.2343 | 24.0144 | 26.8305 | 1.0379 | 3.6029 | 1.1932 | 1.3165 |
| 18 | 25.2059 | 18.6768 | 24.3389 | 27.1793 | 1.0013 | 3.5902 | 1.1502 | 1.2033 |
| 19 | 25.7802 | 19.6234 | 24.9986 | 27.8286 | 0.9268 | 3.5618 | 1.0648 | 0.9799 |
| 20 | 26.4485 | 20.8146 | 25.7676 | 28.4758 | 0.8401 | 3.5235 | 0.9690 | 0.7378 |
| 21 | 26.8054 | 21.4962 | 26.1801 | 28.7717 | 0.7938 | 3.5003 | 0.9193 | 0.6191 |
| 22 | 28.3427 | 24.8999 | 27.9893 | 29.6516 | 0.5944 | 3.3693 | 0.7157 | 0.2226 |
| 23 | 28.5952 | 25.5512 | 28.2950 | 29.7388 | 0.5617 | 3.3413 | 0.6836 | 0.1778 |
| 24 | 28.7821 | 26.0541 | 28.5234 | 29.7941 | 0.5374 | 3.3189 | 0.6601 | 0.1484 |
| 25 | 28.9941 | 26.6486 | 28.7854 | 29.8481 | 0.5099 | 3.2917 | 0.6336 | 0.1189 |
| 26 | 29.2767 | 27.4843 | 29.1395 | 29.9065 | 0.4733 | 3.2520 | 0.5987 | 0.0858 |
| 27 | 29.4055 | 27.8831 | 29.3032 | 29.9285 | 0.4566 | 3.2324 | 0.5828 | 0.0730 |
| 28 | 29.5674 | 28.4022 | 29.5111 | 29.9523 | 0.4356 | 3.2063 | 0.5630 | 0.0587 |
| 29 | 29.7872 | 29.1411 | 29.7980 | 29.9786 | 0.4071 | 3.1679 | 0.5362 | 0.0426 |
| 30 | 30.0271 | 29.9973 | 30.1177 | 30.0002 | 0.3759 | 3.1215 | 0.5071 | 0.0288 |

### 3.4.3 Reliability Function ($\hat{R}(\tau)$)

In this work, to analyze reliability performance, the reliability property was analyzed by inputting the mission time ($\tau$) after the final failure time ($x_n = 187.35$H) cited in the previous section. Here, reliability function is well known as a technique for analyzing the reliability of a software system model by inputting the mission time.
Therefore, the equation for measuring reliability ($\hat{R}(\tau)$) is as follows [19].

$$\hat{R}(\tau|x_n) = exp[-\{m(x_n+\tau) - m(x_n)\}] \quad (33)$$

Figure 5 shows a graph analyzing the reliability after inputting random mission times (1 to 145H) into the model presented in this study. In general, the higher the reliability, the better the reliability of the model. Therefore, as shown in Figure 5, the larger and more stable the reliability value of the analyzed model, the better the reliability of the model

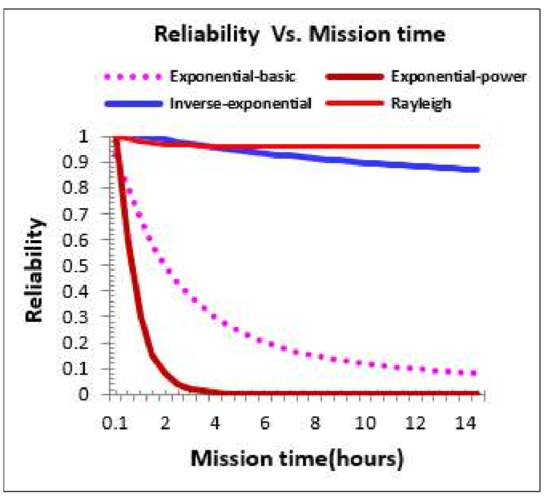


*Figure 5: Result of $\hat{R}(\tau)$ Analysis.*

Therefore, the reliability of the Rayleigh and Inverse-exponential models is efficient as it is stable and shows high reliability values as mission time passes. However, the reliability properties of the Exponential-basic and Exponential-power models showed reliability values that continued to decrease over time. In other words, it can be seen that it is an inefficient model.

Table 5 shows data analyzing the trend of reliability values in detail by mission time invested to analyze the properties of the $\hat{R}(\tau)$ function that determines reliability performance. Additionally, to facilitate data analysis, the quoted mission times were converted to 1/10 (mission time$\times 10^{-1}$).

*Table 5: Analysis of Reliability Function.*

| Mission Time (hours) | $\hat{R}(\tau)$ | | | |
|---|---|---|---|---|
| | Exponential-basic | Exponential-power | Inverse-exponential | Rayleigh |
| 0.1 | 0.9271 | 1.0274 | 1.0187 | 0.9972 |
| 0.5 | 0.8022 | 0.5924 | 1.0110 | 0.9877 |
| 1 | 0.6764 | 0.3001 | 1.0018 | 0.9791 |
| 1.5 | 0.5764 | 0.1533 | 0.9932 | 0.9730 |
| 2 | 0.4962 | 0.0790 | 0.9850 | 0.9688 |
| 2.5 | 0.4312 | 0.0411 | 0.9773 | 0.9659 |
| 3 | 0.3780 | 0.0216 | 0.9700 | 0.9639 |
| 3.5 | 0.3341 | 0.0114 | 0.9630 | 0.9625 |
| 4 | 0.2977 | 0.0061 | 0.9564 | 0.9616 |
| 4.5 | 0.2671 | 0.0033 | 0.9502 | 0.9610 |
| 5 | 0.2413 | 0.0018 | 0.9442 | 0.9606 |
| 5.5 | 0.2194 | 0.0010 | 0.9386 | 0.9603 |
| 6 | 0.2007 | 0.0005 | 0.9331 | 0.9602 |
| 6.5 | 0.1846 | 0.0003 | 0.9280 | 0.9600 |
| 7 | 0.1707 | 0.0001 | 0.9230 | 0.9600 |
| 7.5 | 0.1586 | 0.0001 | 0.9183 | 0.9599 |
| 8 | 0.1480 | 0.0006 | 0.9138 | 0.9599 |
| 8.5 | 0.1388 | 0.0000 | 0.9094 | 0.9599 |
| 9 | 0.1306 | 0.0000 | 0.9052 | 0.9599 |
| 9.5 | 0.1234 | 0.0000 | 0.9012 | 0.9599 |
| 10 | 0.1170 | 0.0000 | 0.8974 | 0.9599 |
| 10.5 | 0.1114 | 0.0000 | 0.8937 | 0.9599 |
| 11 | 0.1063 | 0.0000 | 0.8901 | 0.9599 |
| 11.5 | 0.1017 | 0.0000 | 0.8867 | 0.9599 |
| 12 | 0.0977 | 0.0000 | 0.8834 | 0.9599 |
| 12.5 | 0.0940 | 0.0000 | 0.8802 | 0.9599 |
| 13 | 0.0907 | 0.0000 | 0.8771 | 0.9599 |
| 13.5 | 0.0876 | 0.0000 | 0.8741 | 0.9599 |
| 14 | 0.0849 | 0.0000 | 0.8713 | 0.9599 |
| 14.5 | 0.0824 | 0.0000 | 0.8685 | 0.9599 |

### 3.5. Step 5: Presentation of Performance Evaluation Data Needed by Software Developer.

The subject of this study was the analysis results of model efficiency (MSE, $R^2$) and attribute data (m(t), λ(t), R(t)) that have a significant impact on model performance.

Table 6 shows the final evaluation results after comprehensively comparing the performance attribute data. That is, according to the analysis results, it was concluded that the Inverse-exponential model had the best performance. For software developers, this result can be used not only as basic design data required in the early stages, but also as data to improve reliability needed to solve defects.

*Table 6: Evaluation Result of the Proposed Model*

| NHPP model | Model Efficiency | | Model Performance | | |
|---|---|---|---|---|---|
| | MSE | $R^2$ | m(t) | λ(t) | $\hat{R}(\tau)$ |
| Exponential-basic | Good | Good | Good | Worst | Worst |
| Exponential-power | Best | Best | Best | Worst | Worst |
| Inverse-exponential | Best | Best | Good | Best | Best |
| Rayleigh | Good | Good | Good | Best | Best |

## 4. CONCLUSION

Software developers must be able to collect failure time data of software systems that are already operating normally in the field they wish to develop and then apply this to the initial development process to prevent possible failures in advance. Also, if developers can resolve the cause of failure before releasing the software, they will be able to improve the performance of the software by exploring reliability attributes more efficiently. Accordingly, the performance of the proposed reliability model was newly explored by applying the failure time data required by developers to the software reliability model with exponential-type distribution properties, and the related attribute data was analyzed in detail.

The results of this study are as follows. First, as a result of analyzing the MSE and $R^2$, which are the standard values for selecting an efficient model, the proposed models were all found to be efficient with a performance of over 85%.

Second, in trend analysis of true value using the m(t) function, which has an important impact on the subject of this study, the Exponential-power model was efficient. Also, in the analysis of the occurrence intensity of instantaneous failure using the λ(t) function, the Inverse-exponential and Rayleigh models were efficient as the failure rate showed a trend of decreasing over time.

Third, as a result of measuring future reliability, the Inverse-exponential and Rayleigh models were efficient as they showed high reliability and stable trends over time. Thus, as a result of detailed analysis of performance attribute data (MSE and $R^2$, m(t), λ(t), $\hat{R}(\tau)$), the performance of the Inverse-exponential model was the best.

In conclusion, this study can present algorithmic techniques and basic design data that can analyze and predict performance attribute data needed by developers during the early software development process. Additionally, follow-up research will be needed to use the results of this study to find an optimized reliability model suitable for related software application fields and to explore attribute data related to reliability performance.

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