

CAAMP NET: AMP BASED CBAM ATTENTION NETWORK FOR CS RECOVERY

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ABSTRACT

Deep learning based compressed sensing is a novel technology that has led to complete signal recovery with fewer measurements that are well below the required number of samples defined by Nyquist rate. It outperforms the existing non-deep learning-based methods by exploiting the inherent structures present in the signal or image of interest. There are iterative deep network translations of state of art recovery methods, as well as standard deep learning architectures in literature. The proposed method uses joint optimization of Compressed sensing as well as recovery using an Approximate Message Passing (AMP) based iterative method. This work also introduces use of attention gates along with AMP based recovery of compressively sensed image. Using attention helps the model to focus on high data areas and faster recovery. Experimental results show that joint optimization as well as using attention mechanism in the model outperforms other Deep learning-based CS recovery methods in a range of measurement rates.

Keywords: *Approximate Message Passing, AMP, Attention, Compressed Sensing, Compressive sensing, CBAM Attention*

1. INTRODUCTION

Compressed sensing enables robust image recovery with faster sensing through a set of random measurements from the image [1,2]. It has an extensive set of applications in places where sensing is costly and/or time consuming. Real time applications of compressed sensing can be seen in Fast MRI [3], Satellite imaging, channel estimation and Image encryption. Compressed sensing provides strong mathematical guarantees for recovery of the images even in presence of noise [1]. However, there are some prerequisites to be satisfied such as sparsity of the image in some domain. The main challenges in utilizing compressed sensing technology are

- Recovery Complexity - Complexity and time taken for recovery of original image
- Sparsity constraint - Image/ signal is sparse in known domain.

In recent times, deep learning has addressed some of these limitations. Once the deep learning model is trained for the set of images, the recovery time is decreased to a few seconds. Due to inherent learning on patterns from a set of similar images during model training, sparsity constraint and constraints on minimum required measurements for recovery are relaxed.

Traditional CS recovery problem is solved by many iterative methods like iterative soft thresholding [8], AMP (Approximate message passing) [9] and ADMM (Alternative direction method of multipliers) [18]. Many iterative methods employ exploiting some image prior along with the algorithm. For example, some of the recent methods exploit Non local priors, Bayesian priors [10, 11]. These are non-neural network-based recovery algorithms that has high complexity at recovery end. In recent times, the problem of CS image recovery actively adapts deep learning by using Neural network translations of the iterative methods. For example, popular algorithm ISTA is translated to Neural network as ISTANET [13], ISTA net++, EALISTA [19]. Some methods employ direct neural network (NN) implementations to solve CS recovery problem. MD-Recon-Net [20], CSNET+ [21], CNN With Attention [22] are some of the CNN based CS solvers.

To capture additional information, various approaches employ complex networks to formulate a suitable regularizer [13]. However, the regularizer's inherent uncertainty and indescribability make direct learning challenging. Consequently, rather than learning the regularizer itself, associated information is acquired, such as the

denoising prior, and the gradient. Nonetheless, the extensive parameter count typical of traditional deep neural networks restricts the practical deployment of these models. This paper introduces a deep unfolding model that unravels the iterative noise

reduction process, viewing it through the lens of the AMP algorithm's denoising strategy.

To accommodate multiple sizes of images and generalize it, blocked based image recovery

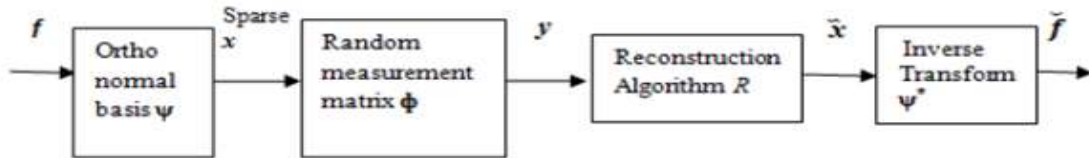


Figure 1: Block Diagram representing Compressed sensing

algorithms are proposed in literature. OPINE NET [23], BCS Net [24], DR2 Net [15] implements block-based image recovery based on different iterative algorithm implementations.

Recently, optimization of the sampling matrix has been integrated into certain model-based approaches and traditional deep neural networks [21], which are tailored for image compressed sensing (CS). This technique involves training the sampling matrix alongside other parameters through gradient descent-related algorithms. Different types of sampling matrices are created based on the optimization constrains. This paper concentrates on the floating-point matrix in the trained model. Using floating point matrix helps normalization and gives the accuracy compared to binary sampling matrices. Given the critical role of the sampling matrix in the sampling and reconstruction phases of most deep unfolding methods [13] selecting an optimal sampling matrix can significantly enhance reconstruction quality. In our research, we train the sampling matrix in conjunction with other parameters within our proposed deep unfolding framework.

Within this context, it becomes imperative to enhance the current methodologies documented in scholarly works to efficiently address the challenges posed by visual compressed sensing. The envisioned model ought to possess the capability to reconstruct compressed sensing images from a reduced number of measurements while maintaining a satisfactory level of recovery quality.

As part of our work, a novel deep learning based compressive sensing algorithm is proposed in the paper to solve the visual compressed sensing problem. Our method not only defines a dubbed AMP based recovery network, but also utilizes attention such that the recovery network will refine the recovery by concentrating on most significant parts of the image block. A joint optimization of sensing and recovery showed that the resulting model is considerably outperforming some of the

state of art recovery techniques. Our significant contributions through the paper are as follows:

1. Developed a CBAM Attention based Approximate Message Passing Network (dubbed as CAAMP Net) Model, based on AMP (Approximate Message Passing) algorithm. Calculating AMP based noise term in a CNN network gives better noise estimation.
2. A CNN based module is proposed for deblocking, to exclude blocking artifacts raised due to block based reconstruction.
3. A CBAM Resnet (Convolution Block Attention Module with Residual network) is used in combination with AMP network. To our knowledge, this method is the first work to utilize CBAM based self-attention in combination with AMP recovery. The proposed network also takes the output of the attention module as a skip connection to AMP block output. The weighted summation is used to get the recovered output. Because of this, the network converges faster.
4. In CAAMP Net, Sampling matrix and weighted attention addition parameters are learned as part of the network training and optimization. Due to this approach, the sampling matrix obtained is data driven and performs better.

This work presents a model that is

The organization of rest of the paper is as below. Next section introduces compressed sensing and its recovery methods. Section 3 explores the recent related works. Section 4 deep dives into AMP algorithm, its neural network translation. It also elaborates attention, CBAM and how the CAAMP net is constructed in total. Section 5 compares the experimental results to other state of art networks. Section 6 gives the conclusion and future scope of the paper.

2. BASICS OF COMPRESSED SENSING

Consider an image f of size $N \times N$ is sparse in some ortho-normal basis ψ such that that $f = \psi x$. There

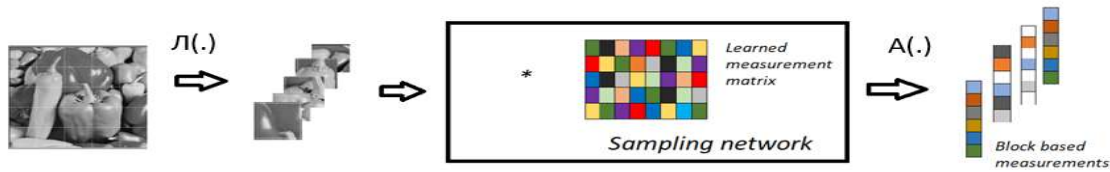


Figure 2: Representation of sampling network

are M random measurements taken from sparse x , where $M \ll N$. Using the resultant measurements vector y , first the sparse approximation \tilde{x} is reconstructed and the original image can be reconstructed back as \tilde{f} . Figure 1 shows a high-level block diagram of compressed sensing. Cades et.al There are a wide variety of algorithms for CS recovery in literature. There are convex optimization algorithms [5], Greedy algorithms [6, 7], thresholding algorithms [8, 9] and non-convex approaches like Bayesian approach [10, 11] are traditional CS recovery methods. In recent times Neural network-based recovery algorithms [12, 13], Deep learning-based recovery algorithms [14, 15] are becoming popular for fast and reliable CS recovery. [16] outlines different CS recovery algorithms and their comparison.

$$\hat{x} = \underset{x}{\operatorname{argmin}} \|x\|_1; \text{ s.t. } \frac{1}{2} \|(y - \psi x)\|_2^2 \leq \epsilon(1)$$

In neural network-based recovery methods, there are majorly 2 types of networks. The traditional iterative recovery algorithms are dubbed to multiple layers of neural networks and trained for optimum results. Another type of networks simply employs direct deep learning based neural networks for direct reconstruction. Some of the recent applications of CS do not require reconstruction and simply require inference based on compressive measurements. The recovery algorithm followed in this paper is implementation of iterative algorithms using Neural networks. AMP (approximate message passing) algorithm assumes the recovery term in each iteration is composed of signal and noise term, and signal is recovered by iteratively denoising the interim recovered image at each stage. This is referred as the context of denoising proposed in the AMP algorithm [17].

3. RELATED WORKS

Deep unfolding techniques merge the strengths of both model-based approaches and traditional deep learning networks. For instance, the well-known ISTA algorithm is expanded into ISTA-Net [13], frequently utilized for addressing optimization challenges that involve a sparsity-promoting regularizer. This method utilizes Convolutional Neural Networks (CNNs) to ascertain the necessary transformation procedures and employs adjustable soft thresholding operations that capture

[4] showed that l_1 and l_0 minimizations are equivalent as long as certain conditions like RIP (Restricted Isometry property) are met for random measurement matrix. Equation 1 shows the recovery methodology by l_1 minimization.

the data's sparse nature. NL-CSNet [25] utilizes non local priors for CS recovery. NN translation of ADMM is implemented by ADMM-CSNET [26]. The advantages of Neural network-based recovery networks are, model training inherently picks up some of the important features as image priors and derive patterns in it. So, the recovery through a trained network is much more faster and result in better recovery compared to non-neural network implementations. Some of the NN implementations consider the priors in transformation domain and implement multiscale CS. CMS Net [9], MS DCI [27], wavelet based [28] are implementations based on transform domain. CASNET [29] utilizes a lightweight CNN to create a feature map and uses it to allocate different measurement rates per block based on the information contained in each block.

As the measurement allocation is based on information presence, important data will be preserved during measurement process. There are some NN implementations of Denoising perspective inspired AMP like LDAMP [30] and AMP net [12]. LDAMP introduces an Onsager correction term unlike the proposed method. AMP-Net utilizes Denoising perspective of AMP and also utilizes deblockers to get past the blocking artifacts.

4. CAAMP NET

In this section, the details of CAAMP Net are presented. Figure 3 shows the block diagram of the proposed CAAMP Net. The model consists of 2 stages. First stage is a Sampling Network and second stage is a reconstruction network. Reconstruction network in turn consists of 2 stages. 1st stage has a set of stacked denoisers along with deblocks and 2nd stage consists of CBAM Resnet module. Weighted sum of the outputs of Denoisers and CBAM RESNET produces the reconstructed image

4.1 Sampling Network

Figure 2 shows block diagram of sampling network. Let function $\Gamma(\cdot)$ denotes the splitting function which creates blocks of equal sizes from the given single channel image, by splitting it into non

overlapping blocks similar to the one in CSNet [21]. As a result, the N*N image is converted into a series of n*n blocks. Equation 2 shows the representation of X into n*n blocks.

$$X_i = \Gamma(X, n) \quad (2)$$

where $i \in \{1, 2, 3, \dots, \frac{N^2}{n^2}\}$. At the end of recovery, the reverse operation of combining the non-overlapping blocks is represented by $\Gamma^{-1}(\cdot)$, such that

$$X = \Gamma^{-1}(X_i, n) \quad (3)$$

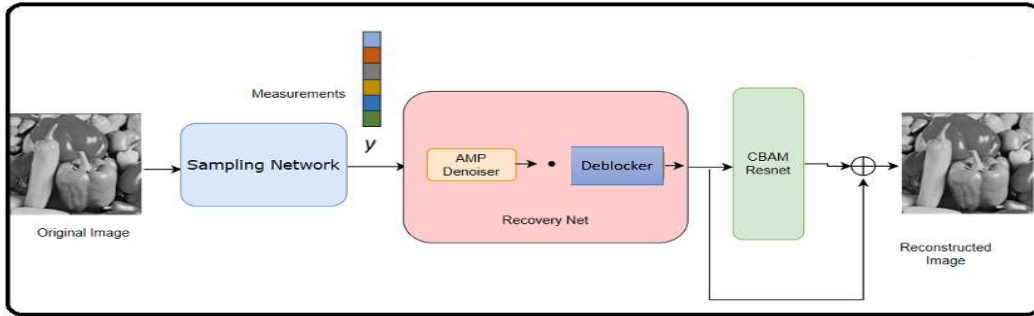


Figure 3: Block diagram of CAAMP Net

Each block X_i is then vectorized $\text{Vec}(\cdot)$ and random measurements are taken from measurement matrix $A(\cdot)$. In the initializing phase, coefficients of sampling matrix are randomly generated. By the end of training process, coefficients of sampling matrix are learned.

The output of Sampling Model is a set of vectors Y which contains measurements from all blocks using measurement matrix A.

$$Y = \text{Vec}(\Gamma(X, n)) * A \quad (4)$$

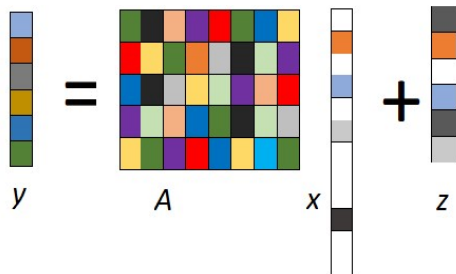


Figure 4: Representation of measurement process

4.2 Reconstruction Network

Figure 3 shows the block diagram of CAAMP net including the reconstruction Model. Reconstruction model consists of series of Denoisers followed by deblocks inspired by AMP NET [12] and CBAM based Resnet for self-attention.

4.2.1 AMP Based Recovery

The proposed method is dubbed neural network version of AMP (approximate message passing) algorithm. In an ideal world the set of measurements obtained from an Image f is Y. The sparse

representation of f in ψ domain is x such that $f = \psi * x$. Taking M random measurements from X using a measurement matrix ϕ with result in equation 5

$$y = \phi * f = \phi * \psi * x = Ax \quad (5)$$

In all practical applications, there will be a noise term added while taking the measurements as shown in equation 6

$$y = Ax + z \quad (6)$$

The problem of estimating x is translated to a denoising problem in the following way.

X is repeatedly estimated, by calculating the pseudo data corresponding to each location in x.

$$r^t = y - Ax^t \frac{r^{t-1}}{M} \langle \eta_{t-1}'(x^{t-1} + A^T r^{t-1}) \rangle \quad (7)$$

$$\text{Pseudo data } \vartheta^t = x^t + A^T r^t \quad (8)$$

$$\text{Denoising } x^{t+1} = \eta_t(\vartheta^t) \quad (9)$$

Approximate Message passing iteratively estimates signal X from measurements Y by estimating the noise term Z, and passing the noise to subsequent layers [9]. Equations 7 8 9 shows calculation of residual at each iteration, followed by

pseudo data calculation and then denoising to obtain denoised version of x . Figure 4 shows how measurements are taken and Figure 5 shows how iteratively scalar term is calculated in AMP algorithm. Correction term in equation 7 ensures that, the noise terms does not get correlated with iteratively estimated X and also ensures that noise term is a Gaussian distribution.

Taking denoising perspective of AMP, an iterative Neural network model is constructed as part of Recovery Net taking inspiration from [12]. Denoising is done by a CNN of 3 layers, where each layer is combination of 2D convolution and ReLU activation function. Denoiser is represented by Π_t where t represents the iteration of expanded

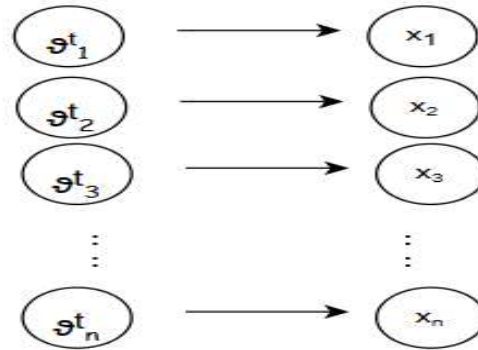


Figure 5: Scalar data calculated in each iteration

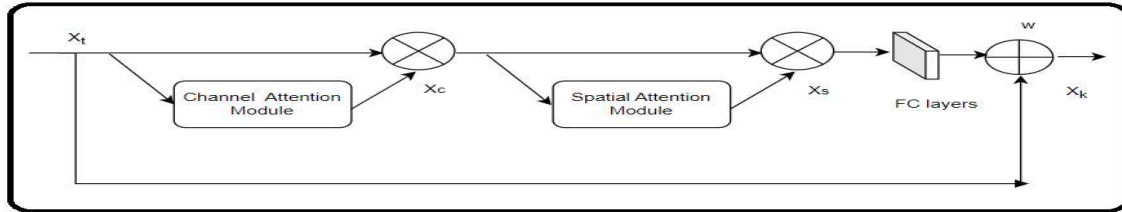


Figure 6: Structure of CBAM Resnet

AMP implementation. As part of training process, input vector is transformed in number of ways, so that the resulting learned model is robust to shift/ transformation and noise. As part of this algorithm, the adjustment term in equation 7 is obtained by calculating $(\alpha A^T A - I) \Pi_t(X_{t-1})$ where α is the learned correction term.

4.2.2 CBAM Resnet

CBAM stands for Convolution Block attention Module. Attention module enable CNN to focus more on more important parts of image than background information. The output of the any attention module is a feature map, that shows significant parts of the map. In CBAM attention, both channel attention and special attention are calculated respectively.

Figure 6 shows the CBAM ResNet module structure in stage2. First, Channel attention is calculated as shown in Figure 7 that gets the most important channel of the feature map. Channel attention Module is represented by M_c and output of Channel attention is represented by X_c . Channel attention is obtained by combination of average pooling and max pooling output passed through multi-layer perceptron (MLP), created by 2 layers of Convolution 2D with ReLU activation function between them. The output is then passed through a Sigmoid function to get most significant channel in the feature map.

The output of channel attention is then passed to special attention module that gets which parts of the feature map contains most significant information. Figure 8 shows the structure of special attention module created by set of convolutions 2D layers followed a sigmoid activation function.

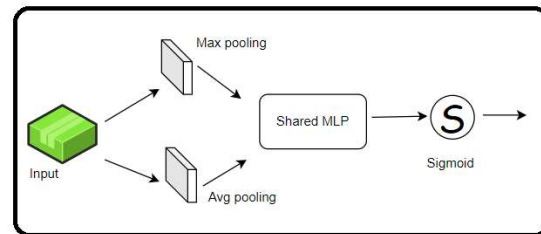


Figure 7: Representation of Channel attention

The output is then passed through Feedforward network realized by 3 layers of Conv 2D networks with ReLU activation. Spatial attention is represented by M_s and output of spatial attention module is represented by X_s . The output is then passed through F_c , feed forward network producing output X_f .

Considering intermediate feature map $F \in R^{C \times H \times W}$ as input, CBAM sequentially infers a 1D channel attention map $M_c \in R^{C \times 1 \times 1}$ that contains most significant information and a 2D spatial attention map $M_s \in R^{1 \times H \times W}$ that contains most significant special information. The result is then used in Resnet to get attached to the original output.

Using attention as part of a deep learning technique enhances the performance of the network.

By adding weighted residual addition of X_t with learned weight W , the output converges faster.

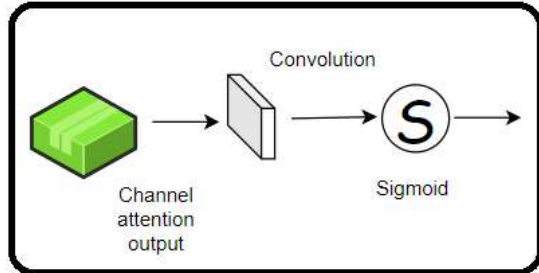


Figure 8: Representation of Spatial attention

4.2.3 Training and Loss function

Algorithm1 shows the forward pass of training of CAAMP net. $N_\alpha, N_\sigma, N_\phi, w, K$ shows the set of trainable parameters. $N_\alpha = \{\alpha_1, \alpha_2, \alpha_3 \dots\}$ where α is adjustment parameter for residue term. N_σ, N_ϕ represents parameters for denoiser and CBAM modules respectively. W represents weighted summing parameter learnt as part of training. Along with these parameters A and B , sampling and initialization matrices are also learned as part of training process. Loss function is calculated as Mean squared error between X and X_k for each image block.

$$L = \frac{1}{\left(\frac{N^2}{n^2}\right)} \|X_n - X_n^K\|^2 \quad (10)$$

Algorithm: CAAMP Net Model -forward propagation

Input: $X, A, B, N_\alpha, N_\sigma, N_\phi, w, K$

Output: X^k

Sampling:

$$Y = A \cdot \text{Vec}(\Gamma(X, n))$$

Reconstruction:

- 1: Set $t = 0$ #initilize
- 2: $X^t = \Gamma^{-1}(\text{vec}^{-1}(B * Y))$ # initial estimate
- 3: for $t < K$ do
- 4: $t \leftarrow t + 1$
- 5: $\vartheta^t = (\alpha_t A^T A - I)$
- 6: $r^{t-1} = Y - A(\text{vec}(\Gamma(x^{t-1}, n)))$ # residue
- 7: $X^t = \Gamma^{-1}(\text{vec}^{-1}(\alpha_t A^T r^{t-1} + \text{vec}(\Gamma(X^{t-1}, n)) - \vartheta^t \text{vec}(\Pi_t(\Gamma(X^{t-1}, n))), n))$
- 8: $X^t = X^t - B(X^t)$
- 9: $X^c = M_c(X^t)$
- 10: $X^s = M_s(X^c * X^t)$

$$11: X^a = X^c * X^s$$

$$12: X^f = F_c(X^a)$$

$$13: X^k = X^f + w * X^t$$

$$14: \text{return } X^k$$

Algorithm 1: Forward propagation of CAAMP NET

5. CAAMP NET

Figure 3 shows the block diagram of the network where it has a Sampling Net, a Reconstruction Net and an Attention Net.

The sampling net employs basic sampling based on number of required measurements. A block-based sampling (33*33) blocks is used to take measurements on the image. Block based imaging ensures that the method works with variations in size of the image. Computational resources required are limited. As part of Reconstruction Net, AMP net is implemented by a denoiser Module (Made up of a CNN model) followed by a deblocking module (A CNN based network) is utilized iteratively to improve reconstruction quality. Each CNN consists of four convolutional layers with a filter size of (3 X 3). The first three convolution layers include a bias term and are followed by a ReLu activation function. The fourth layer does not have a bias term. The input/output channels for all four layers are 1/32, 32/32, 32/32, and 32/1, respectively.

In the attention network, a CBAM based residual attention module is used to estimate the most important features of the current block and a weighted addition is done with the output of the deblocked module.

CBAM RESNET is made up of a Bottleneck module, CBAM module, a feedforward network and then a down sample network.

Bottleneck module is implemented by 3 layers of 2 dimensional convolutions, each followed by ReLu activation function. input/output channels are 1/32, 32/32, 32/1 respectively. Kernel size for all the layers is 1X1.

A CBAM Module has both channel and spatial attentions in sequence. channel attention is implemented using average and max pooling followed by 2 convolution layers with stride as 1X1 and kernel size as 1X1. Input/output channel are 64/4 and 4/64 respectively. Spatial attention module is implemented by single Convolution layer with Kernel size as 7X7, stride as 1X1 and padding as 3X3. followed by Sigmoid function.

The feed forward network consists of 3 convolution layers. First 2 layers have ReLu activation and the last layer don't have any activation function.

The input/output channels are 1/64, 64/64, 64/1 respectively. c

Finally, the down sampler has 2 convolutional layers with input/output channels are 64/32, 32/1 respectively. Kernel size for both the layers is 1X1.

This setting is repeated for N number of layers and the output images of each image block is joined back to form the recovered image. In the current setting a 6-layer blocks of network are used to evaluate the performance. Sampling matrix and weighted attention addition parameters are learned as part of the network training and optimization

6. RESULTS AND DISCUSSION

BSD 500 data set is used for training the model. Testing is done using set 11 data set. As part of training phase, each image in BSD500 data set is changed into blocks of 33*33. 200 images are used as training set and next 100 images are used as testing set in each epoch. A set of 100 epochs were done to obtain optimum model parameters in this setting. The resultant model is then tested against Set 11 with different CS sampling ratios. The results are compared against other 4 state of art recovery techniques.

Table 1 shows the results of CS recovery over wide range of measurement rates. It shows that, proposed CAAMP method outperforms averaged PSNR/SSIM for other 4 CS recovery techniques for most of the measurement rates for Set11. Apart from the numeric results that we can see the feature comparison of different recovery algorithm as part of Table 2.

Along with Trainability, learning of sampling matrix, interpretability, CAAMP net also provides Deblocking and attention consideration features in comparison with other methods. The explainability of the network makes a huge difference to understand the model behavior and also provides ways to improve the same.

Figure 9 and Figure 10 shows visual results of using CAAMP net for camera man image and Building image for various measurement rates and how the fine details are preserved at each measurement rate. With even 10% measurements, most of the features of the image are preserved, however 30% measurement rate is preserving most of the features without considerable visual degradation.



Figure 9: Cameraman recovered with CAAMP Net (PSNR (dB)/SSIM) with 30%, 25% and 10% measurement rates

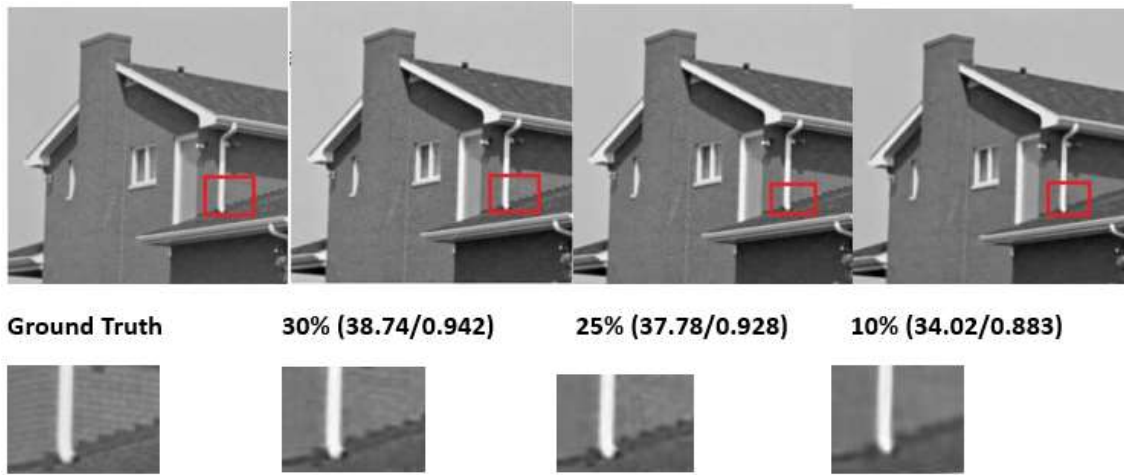


Figure 10: Reconstruction results of building image using CAAMP Net (PSNR (dB)/SSIM) for various measurement rates

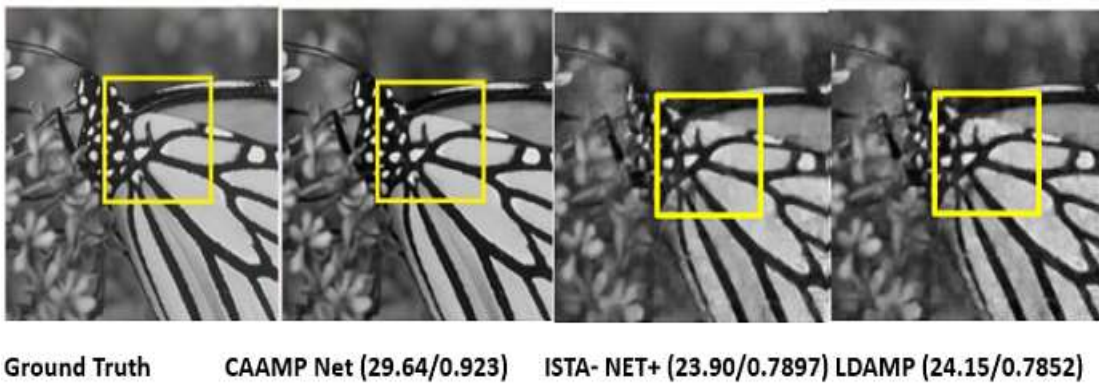


Figure 11: Reconstruction results of Monarch Image (PSNR (dB)/SSIM) at 10% measurement rate

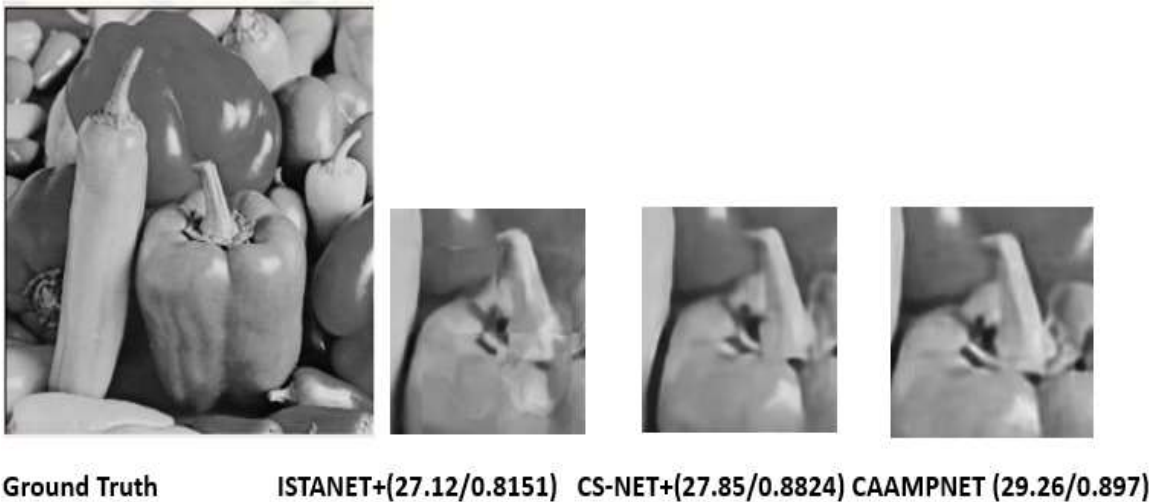


Figure 12: Peppers recovery comparison (PSNR (dB)/SSIM) with 10% measurement rate

Table 1: Results of average PSNR/SSIM ratio using different recovery methods on Set 11

Method	10% measurement (PSNR/SSIM)	25% measurement (PSNR/SSIM)	30% measurement (PSNR/SSIM)
CAAMP-NET	<u>29.2745/0.8767</u>	34.3025/0.9449	35.8044/0.9574
NL-CSNet (IEEE 2021)	30.05/0.8995	<u>34.12/0.9440</u>	<u>35.68/0.9606</u>
ISTA-NET+ (IEEE 2021)	25.93/0.7840	32.27/0.9167	33.66/0.9330
CS Net+ (TIP 2020)	28.37/0.8580	32.76/0.9322	34.30/0.9490
LDAMP(NIPS 2017)	24.94/0.7483	29.93/0.8783	32.01/0.9144

Table 2: Comparison between features of CAAMP net with others

Method	Trainability	Sampling matrix learnability	Interpretation	Deblocking	Attention consideration
NL-CSNet (IEEE 2021)	√	×	×	√	×
ISTA-NET+ (IEEE 2021)	√	√	√	×	×
CS Net+ (TIP2020)	√	×	×	×	×
LDAMP (NIPS 2017)	√	×	√	×	×
CAAMP-NET	√	√	√	√	√

Figure 11 and Figure 12 shows the visual result comparison for different measurement rates for proposed CAAMPNET with other state of art methods. For 10% measurement rate, CAAMP net clearly preserves most of the image features better than other compared methods.

Currently the proposed network allocates equal number of measurements across all the blocks. This method can be improved on adaptive rate allocation (ABCS). The method also can be trained with different families of images as the attention network can improve the adaptability.

6. CONCLUSION AND FUTURE SCOPE

A Novel deep unfolding network CBAM ResNet based AMP NET (CAAMPNET) is proposed inspired by denoising lens of AMP algorithm. Unlike other AMP inspired algorithms, the proposed method also employs Attention ResNet for CS recovery. CBAM ResNet uses CBAM attention to pick up the most significant channel and also most significant spatial information in this channel. Sampling matrix and denoiser parameters are learned as part of model training. This helps in Weight parameter of attention module is also learned as part of training phase.

This method facilitates the integration of attention mechanisms within unrolled neural

network architectures. This synergistic combination leverages the interpretability inherent in AMP-based networks while also benefiting from the autonomous learning capabilities of neural networks. The result is a powerful framework that not only provides clarity in its operations but also adapts and improves through self-attention. Results of our experiments demonstrate that the CAAMPNET algorithm improves the results of other Deep learning based Compressed Sensing recovery algorithms.

In future, versatility of this network over different families of images, like natural images/ medical images is tested since use of self-attention helps in understanding most significant and important features within the image. The proposed method can be further improved by using rate allocation per block based on the presence of information, rather than allocating same number of measurements for the block. The proposed algorithm can also be improved for robustness by training the model with noisy images.

Availability of supporting data: Implementation of the model details will be shared upon reasonable request. Data sets used for training and testing are standard data sets.

Competing interests: The authors declare that they have no conflict of interest.

Authors' contributions: Both the authors have equal contribution in the work.

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