Journal of Theoretical and Applied Information Technology
15th July 2025. Vol.103. No.13

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

OPTIMIZED RELIABILITY FRAMEWORK FOR SERIES-PARALLEL SYSTEMS: STRATEGIC REDUNDANCY ALLOCATION USING HAM AND IPM SOLUTIONS

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ABSTRACT

When it comes to reliability engineering, the main goal is to make sure that systems and their parts always work as they should within a certain time limit and under certain conditions. Within the domain of reliability theory, system robustness is improved by strategically incorporating redundancy, while simultaneously accounting for constraints such as cost, mass, and spatial requirements in series-parallel architectures. This research focuses on examining how these constraints-specifically weight, volume, dimensional, and spatial factors-influence system reliability enhancement. The investigation is centered on spare parts utilized in a representative Automation Forum setup, where factors like cost, weight, and size are vital to sustaining effective functionality. Unlike electronic systems that may not emphasize these constraints, the Automation Forum includes crucial components such as compressors, condensers, and absorption towers, all of which require thorough reliability evaluation. To address this, the Lagrange multiplier method is employed to design and analyze an integrated redundant reliability model configured in a series-parallel structure. This approach yields continuous-valued outputs for essential variables including component count, individual and stage reliabilities, and total system reliability. To derive feasible and practical integer-based solutions, the study further implements heuristic strategies and integer programming. These techniques significantly enhance the precision and relevance of the reliability analysis conducted.

Keywords: *IRR Model, Series-Parallel Configuration, System Reliability, LAM Approach, HAM Approach, IP Approach*

1. INTRODUCTION

There are often only two possible states for systems and their parts in conventional reliability theory: operating and failed. The analytical scope is limited since intermediate scenarios are ignored by this binary perspective, despite its critical importance. Because the system and its parts can exist along a continuum of states, the multi-state systems paradigm improves this analysis. With additional alternatives, we can understand reliability better and to a deeper level, taking into account all the different kinds of performance and degradation that occur in the actual world. The A lot of research has gone into the Integrated Redundant Reliability Problem (IRRP) in an effort to make systems more reliable by finding the sweet spot for distributing redundant parts while keeping costs down. Methodologies, issue formulations, and potential solutions are highlighted in this paper that outlines the major accomplishments in this area.

To improve system dependability within linear constraints, Mishra, K. B. (1972), developed a mathematical framework that integrates many phases with parallel-redundancy [01]. The model was reformulated as a saddle point problem utilizing Lagrange multipliers, and Newton's method was employed to resolve the equations, thereby improving computational efficiency.
Building on the initial framework, a multistage decision-making model rooted in the Maximum Principle was devised, providing a simple and readily executable answer that guaranteed convergence while reducing computational strain. Improving component allocation in parallel-series

and series-parallel systems was the goal of Ei-Neweihi, E. (1986) as they sought to increase system reliability through the application of majorization techniques and Schur-convex functions [02]. While series-parallel systems benefited from partial ordering during optimization, parallel-series systems depended solely on component reliabilities for efficient allocation. According to the results, these issues may be recast as integer linear programming models, which, when used with Schur function approaches, can yield both exact answers and insightful predictions in certain settings.

Konak (2004), introduced a novel approach to maximize the minimal subsystem dependability in series-parallel systems by merging integer programming for component allocation with sequential max-min subproblems [03]. This approach successfully balanced subsystem reliability while working with limited resources. Ranjan Kumar et al. (2009), highlighted the limitations of single-level approaches and focused on multilayer redundancy [04]. They improved modular redundancy in series and series-parallel configurations using a hierarchical genetic algorithm (HGA), demonstrating that modular designs provide far better dependability than component-level redundancy. A multi-objective RAP model was developed by Ebrahim Nezhad et al. (2011) with the aim of optimizing system reliability and profitability simultaneously [05]. Their method allowed for more practical and flexible designs by using a combination of active and standby procedures as decision factors.

Murat (2012), improved multi-objective RAP using a decomposition-based approach to ensure exact pareto-optimal solutions [06]. Reliability optimization is crucial in the design and management of industrial systems, as system dependability can be augmented through component enhancement or redundant arrangements. Mohsen Ziaee (2013), tackled the Redundancy Allocation Problem (RAP) by introducing a mixed-integer programming approach for hierarchical series-parallel systems, optimizing dual objectives within various resource restrictions [07]. The model's efficacy was illustrated using a numerical example. In their in-depth study of reliability optimization models, Twum and Aspinwall (2013), highlighted shortcomings in redundancy allocation approaches [08]. They argued for more research into different reliability allocation mechanisms and brought attention to the fact that multi-objective models are scarce and that redundancy-based strategies are common.

Ali Zeinal Hamadani and Mostafa Abouei Ardakan (2014), presented an innovative mixed redundancy solution that integrates both active and standby components inside a single subsystem, hence enhancing conventional RAP methodologies [09]. The issue was analyzed using imperfect switching and k-Erlang TTF distribution, then resolved via a genetic algorithm, showcasing enhanced system reliability through application on a regular test problem. Using compromise programming, Roya Soltani (2015), extended the Reliability Assessment Process (RAP) to supplier selection, improving profitability, warranty term, and reliability [10]. Using a nonlinear programming model that accounts for failure rates based on the number of active components and internal connection expenses, Sharifi Mani and Yaghoubizadeh Mohsen (2015), investigated redundancy allocation in series-parallel systems [11]. To enhance system reliability under practical situations, they employed meta-heuristic techniques, including evolutionary algorithms and simulated annealing, with parameter optimization through response surface methodology.

The focus of Pourkarim Guilani et al. (2017), was on improving the design and dependability of multi-state series-parallel power systems in light of increasing cumulative demand [12]. In parallel, Aghaei et al. (2017), improved k-out-of-n systems by amalgamating redundancy strategies with genetic algorithms and integer programming for enhanced optimization [13]. Florin Leon et al. (2020), examined the appropriate distribution of spare modules in extensive series-redundant systems to attain specified reliability within cost limitations, accounting for both active and standby redundancy [14]. They analyzed cold and warm standby techniques for standby subsystems and utilized the Lagrange multipliers method and Pairwise Hill Climbing for optimization, in conjunction with the evolutionary algorithm RELIVE, to improve performance in large-scale systems.

The development, evaluation, and optimization of a cohesive, integrated redundant dependability system was an uncharted territory; Sridhar Akiri et al. (2021) dug deep into this area. To ensure practical applicability, the system's architecture was first studied using the Lagrange multiplier, which yielded integer solutions to improve dependability through integer and dynamic programming methods [15]. The growing field of systems and software modeling, which includes AI-powered bots, HMIs, and software systems, was explained by Sridhar

Akiri et al. (2022). Their work is helpful for students, academics, and professionals in the field since it provides methods for optimization, simulation, and reliability modeling; it focuses on cost modeling and resource allocation for complex systems [16].

Using Lagrangian multipliers and dynamic programming approaches, Srinivasa Rao Velampudi et al. (2023), integrated superfluous reliability into structured systems. By evaluating factors including size, cost, and load, a heuristic was used to generate an integer answer, which improved the system's efficiency [17]. A numerical example was used to illustrate the findings of the suggested method, which sought to improve system performance by assessing factor efficiencies and phase reliabilities. Using priority-based repair strategies, S. C. Malik et al. (2023), developed a reliability model for a four-unit parallel cold standby system [18]. The system separates the two main units into two categories: phase I and phase II. The auxiliary units are given priority when it comes to repairs. Using semi-Markov processes and regenerating point techniques, we explored a number of reliability metrics with applications in power distribution systems. These metrics included availability, profit function, mean time to system failure, and more. In order to enhance the effectiveness of the system, Srinivasa Rao Velampudi et al. (2024), performed a case study on the Muffle Box Furnace [19]. A United Reliability Model (URM) was developed as a result of the research, which used Lagrangean techniques to determine the weight, volume, and price parameters for different system configurations. Using weight and volume as additional limitations, the study integrated value restrictions into IRR models, highlighting the link between component cost and dependability.

Sukumar V. Rajguru and Santosh S. Sutar (2025), introduced a k-out-of-m load-sharing reliability model featuring unequal load distribution among components, thereby solving a deficiency in current equal-sharing research [20]. Employing the proportional conditional reverse hazard rate model, they generated system failure functions and estimated parameters through maximum likelihood, validated by simulations and empirical data analysis, underscoring practical significance. Ravi Kumar, T. V. Rao, and Sameen Naqvi (2025), examined the best distribution of redundancies in series and parallel systems, taking into account the interdependence of components and redundancies [21]. The work examined stochastic comparisons utilizing copula-based models and generalized distorted distribution functions across different dependency configurations. The findings were substantiated by examples, simulations, and empirical data analysis, offering insights into the strengthening of dependability via dependent redundancy allocation.

Ramadevi Surapati et al. (2025), provided substantial insights into the optimization of series-parallel systems under diverse constraints, while their study mostly focused on traditional oil burner components [22]. However, a shortcoming remains in the application of this methodology to more complex industrial contexts, where factors such as environmental stress, cost efficiency, and adaptive maintenance practices are crucial for system reliability. Moreover, their methodology relied on a two-phase optimization process (Lagrange multipliers followed by dynamic programming), which may not be computationally feasible for large systems. This research aims to address these limitations by introducing an advanced optimization framework that integrates machine learning-based reliability prediction with multi-objective optimization techniques to achieve improved accuracy and flexibility in system reliability assessment.

Ramadevi Surapati et al. (2025), investigated the impact of physical constraints, such as weight, volume, dimensions, and geographic limitations, on the enhancement of system reliability for drilling machine spare components, including pulleys and gears that regulate motion and load [23]. The Lagrangean multiplier approach was employed to develop an integrated redundant reliability model yielding real-valued solutions for component quantities, stage reliability, and system reliability. Heuristic approaches and dynamic programming were employed to develop integer-based solutions aimed at enhancing the analytical precision of dependability assessments. Their study presents a robust foundation for redundancy optimization in series-parallel systems; nonetheless, it focuses static optimization and mechanical systems. Dynamic failure prediction, real-time optimization, and multi-objective decision-making are essential for managing uncertainty, deterioration, and cost-effective maintenance in contemporary reliability engineering. This research incorporates real-time failure monitoring, machine learning-based predictive failure estimation, and an advanced multi-objective optimization framework that equilibrates cost, weight, and maintenance schedules inside the series-parallel reliability model to overcome existing deficiencies. This study enhances the adaptability and practicality of reliability planning across diverse engineering applications by extending the framework to encompass a broader range of mechanical and industrial systems beyond drilling machines.

This study conducted a comprehensive analysis of series-parallel system configurations inside the Integrated Redundant Reliability (IRR) Model, focusing on the impact of redundancy on system reliability. A case study examining critical spare components namely the cooler, vaporizer, and condenser, typically included in Automation Forum systems was conducted. The results provide significant insights for the design and execution of reliability-oriented system designs, tackling practical issues and promoting the use of reliability theory in engineering systems.

The study examined a series-parallel setup by developing an internal rate of return (IRR) model and using the traditional Lagrange multipliers method to find real-valued solutions, taking into consideration both rounded and unrounded results. In order to produce integer values, a novel heuristic algorithm and integer programming methodology were created. This allowed for a comparison with the Lagrangian method and produced conclusions that were based on solid science. Improving the system's overall reliability while preserving the appropriate quantity of components at each phase was the goal of this technique. The study employed a heuristic algorithm and an integer programming methodology to derive discrete values, facilitating a comparison examination with the Lagrangean method and yielding methodologically robust results. The strategy sought to preserve the necessary quantity of components (C_{com}) at each phase while improving the overall system dependability ($R_{sys\ dep}$).

2. APPROACHES:

To find out how well systems and their parts work under certain circumstances, reliability testing is necessary. Elements are assumed to have the same dependability levels within each stage, which means that performance requirements are consistent. Also, the system treats each component as if it were statistically independent, so if one part fails, it won't affect how the other parts work. Components, phases, overall system reliability, and the Integrated Redundant Reliability (IRR) Model are defined below to help with comprehension.

Considerations and Notations:

- Each stage is assumed to consist of identical elements, meaning every element has the same reliability level.
- All elements are considered statistically independent, so the failure of one does not affect the performance of the others in the system.

$R_{sys\ dep}$: Systems Dependability in Series-Parallel Configuration
R_{dep}: Process Phase Dependability $'xy'$, $0 < R_{dep} < 1$
r_{dep}: Component Dependability in the Phase $'y'$; where $0 < r_{Dep} < 1$
C_{com}: A Multitude of Items in Phase $'xy'$
P_{cr} : Price-component of each element in the phase $'xy'$
W_{cr}: Weight-component of each element in the phase $'xy'$
V_{cr}: Volume-component of each element in the phase $'xy'$
$P_{al\ po}$: Maximum allowable price of the component
$W_{al\ wo}$: Maximum allowable weight of the component
$V_{al\ vo}$: Maximum allowable volume of the component
LMA: Lagrange Multiplier Approximation
HM: Heuristic Methodology
IPA: Integer Programming Approach
CRRM: Comprehensive Reliability and Redundancy Model
l_y, m_y, p_y, q_y, u_y, v_y are constants.

3. STRUCTURAL EVALUATION OF THE PROPOSED SYSTEM MODEL

The system's dependability concerning the given value function

$$r_{dep} = \left[e^{\frac{P_{cr}}{l_y}} \right]^{\frac{1}{m_y}} \quad (01)$$

System Dependability to the provided

$$R_{sys\ dep}(t) = \prod_{x,y=1}^{k,n} \left[1 - \Pi\left(1 - R_{xy}\right)\right] \quad (02)$$

From Equation 1,

$$P_{cr} = l_y \ \mathrm{Log}\left(r_{dep}\right)^{m_y} \quad (03)$$

Where P_{cr} is the Price-component dependability.
Similarly, W_{cr} is the Weight-component and V_{cr} is Volume-component can be expressed as

$$W_{cr} = p_y \ \mathrm{Log}\left(r_{dep}\right)^{q_y} \quad (04)$$

$$V_{cr} = u_y \ \mathrm{Log}\left(r_{dep}\right)^{v_y} \quad (05)$$

Since Price-component is linear in C_{com}

$$\sum_{y=1}^{n} P_{cr}\, C_{com} \le P_{al\ po} \quad (06)$$

$$\sum_{y=1}^{n} W_{cr}\, C_{com} \le W_{al\ wo} \quad (07)$$

$$\sum_{y=1}^{n} V_{cr}\, C_{com} \le V_{al\ vo} \quad (08)$$

From 3, 4, 5 we get

$\sum_{y=1}^{n} l_y \, Log(r_{dep})^{m_y} \, C_{com} \leq P_{alco}$ (09)

$\sum_{y=1}^{n} p_y \, Log(r_{dep})^{q_y} \, C_{com} \leq W_{alco}$ (10)

$\sum_{y=1}^{n} u_y \, Log(r_{dep})^{v_y} \, C_{com} \leq V_{alco}$ (11)

The equation modified using the relation

$C_{com} = \frac{Log\, R_{Dep}}{Log\, r_{dep}}$ (12)

$\sum_{y=1}^{n} l_y \log(r_{dep})^{m_y-1} \, Log\, R_{dep} \leq P_{alco}$ (13)

$\sum_{y=1}^{n} p_y \log(r_{dep})^{q_y-1} \, Log\, R_{dep} \leq W_{al\,wo}$ (14)

$\sum_{y=1}^{n} u_y \log(r_{dep})^{v_y-1} \, Log\, R_{dep} \leq V_{al\,vo}$ (15)

The Lagrangian function is formulated as

$$L_{CRRM} = R_{dep} + \sigma_1 \left[\sum_{y=1}^{n} l_y \log(r_{dep})^{m_y-1} \log(R_{dep}) - P_{alco}\right] + \sigma_2 \left[\sum_{y=1}^{n} p_y \log(r_{dep})^{q_y-1} \log(R_{dep}) - W_{alco}\right] + \sigma_3 \left[\sum_{y=1}^{n} u_y \log(r_{dep})^{v_y-1} \log(R_{dep}) - V_{alco}\right] \quad (16)$$

Where $R_{dep},, r_{dep}, \sigma_1, \sigma_2, \sigma_3$ represent ideal points.

$$\frac{\partial L_{CRRM}}{\partial R_{dep}} = 1 + \sigma_1 \left[\sum_{y=1}^{n} l_y \, Tan\, h(r_{dep})^{m_y} \frac{1}{R_{dep}\, Log\, r_{dep}}\right] + \sigma_2 \left[\sum_{y=1}^{n} p_y \, Tan\, h(r_{dep})^{q_y} \frac{1}{R_{dep}\, Log\, r_{dep}}\right] + \sigma_3 \left[\sum_{y=1}^{n} u_y \, Tan\, h(r_{dep})^{v_y} \frac{1}{R_{dep}\, Log\, r_{dep}}\right] \quad (17)$$

$$\frac{\partial L_{CRRM}}{\partial r_{dep}} = \sigma_1 \left[\sum_{y=1}^{n} l_y (m_y - 1) \frac{\log(r_{dep})^{m_y-2}}{r_{dep}} \log(R_{dep})\right] + \sigma_2 \left[\sum_{y=1}^{n} p_y (q_y - 1) \frac{\log(r_{dep})^{q_y-2}}{r_{dep}} \log(R_{dep})\right] + \sigma_3 \left[\sum_{y=1}^{n} u_y (v_y - 1) \frac{\log(r_{dep})^{v_y-2}}{r_{dep}} \log(R_{dep})\right] \quad (18)$$

$$\frac{\partial L_{CRRM}}{\partial \sigma_1} = \sum_{y=1}^{n} l_y \log(r_{dep})^{m_y-1} \log(R_{dep}) - P_{alco} \quad (19)$$

$$\frac{\partial L_{CRRM}}{\partial \sigma_2} = \sum_{y=1}^{n} p_y \log(r_{dep})^{q_y-1} \log(R_{dep}) - W_{alco} \quad (20)$$

$$\frac{\partial L_{CRRM}}{\partial \sigma_3} = \sum_{y=1}^{n} u_y \log(r_{dep})^{v_y-1} \log(R_{dep}) - V_{alco}$$

$$\frac{\partial L_{CRRM}}{\partial \sigma_3} = \sum_{y=1}^{n} u_y \log(r_{dep})^{v_y-1} \log(R_{dep}) - V_{alco} \quad (21)$$

Where σ_1, σ_2 and σ_3 are Lagrangean multipliers.

Employing the LMT, we determine the number of elements in each phase (C_{com}), discover the optimal reliability of components (r_{dep}), compute the reliability of each stage (R_{dep}), and evaluate the overall structural reliability ($R_{sys\ dep}$). The procedure produces an accurate numerical answer for the component's volume, weight, and cost.

4. CASE STUDY PARAMETERS:

This study focuses on optimizing system parameters within an Automation Forum setup, where cost, weight, and volume play a critical role in ensuring overall operational efficiency. Unlike electronic systems, which may not emphasize these constraints, the Automation Forum comprising key components such as the air compressor, condenser, and absorption tower demands precise reliability evaluation. Essential factors like component-level reliability (r_{dep}), stage reliability (R_{dep}), the number of components per stage (C_{com}), and system-wide structural accuracy ($R_{sys\ dep}$) are thoroughly assessed. The research explores optimal component allocation strategies within a series-parallel configuration, incorporating advanced algorithms, redundancy planning, and component replacement methods. To deepen the understanding of reliability in such industrial configurations, a schematic of a typical Automation Forum system is presented in Figure 1.

This work offers a thorough analysis of Series-Parallel configurations in Integrated Redundant Reliability (IRR) Models, with a particular emphasis on redundant reliability related to industrial systems. A case study based on Automation Forum's nitric acid (HNO_3) production process highlights key components like air compressor, condenser, and absorption tower used in converting ammonia (NH_3) and air into nitric acid. These components, costing between \$5,000 and \$70,000 and varying in weight from 500 lbs and 12,000 lbs and volume from 2,000 cm^3 and 10,000 cm^3, are essential to the process and are often arranged in series-parallel configurations to enhance reliability. Absorption Tower, Compressors and Condensers are typically installed in parallel for backup, while the system operates in a series flow to maintain process continuity. The absorption tower may include redundant support due to its critical role. Mathematical methods such as Lagrange multipliers, heuristics, and integer programming are used to model and optimize system reliability, balancing cost, performance, and uptime.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

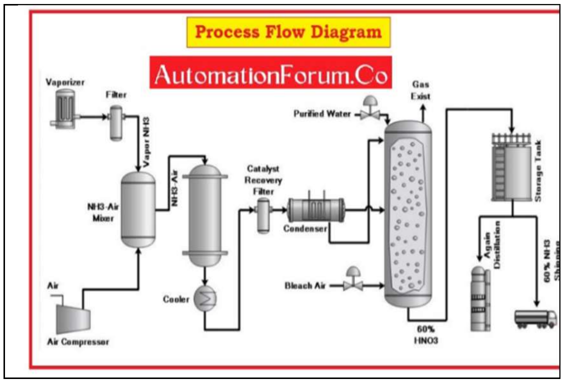


Figure 1: Automation Forum

4.1 Input Design Criteria for the Investigation:

Table 1 contains the necessary constants for the case study.

Table 1: *Establish Fixed Parameters for Cost, Load, and Size in Series-Parallel Configuration Systems*

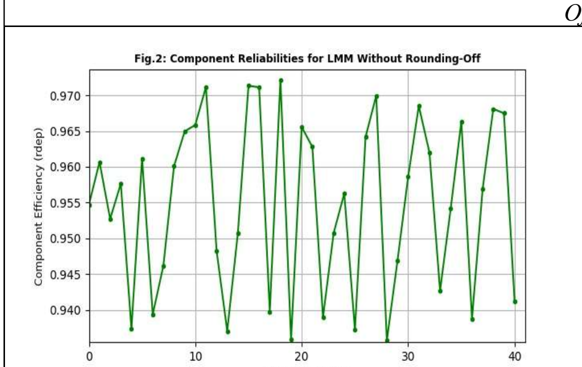
Level	Essential Elements of Value in Currency		*Essential Elements of Load in Pounds*		Essential Elements of Volume in cm^3	
	l_y	m_y	p_y	q_y	u_y	v_y
I	5,000	0.91	500	0.93	2,000	0.92
II	20,000	0.95	5,000	0.95	5,000	0.95
III	40,000	0.98	10,000	0.98	10,000	0.97

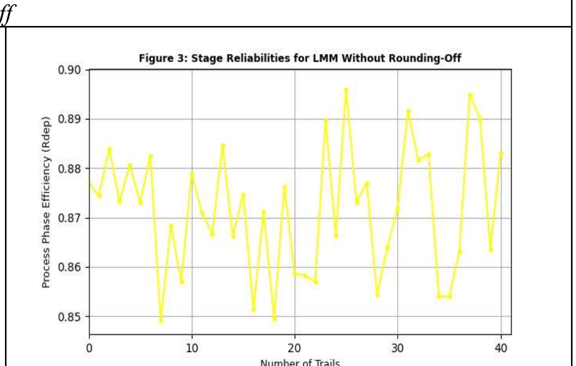
You can see the structural efficiency, along with the efficiency values for each component, phase, and quantity of elements at each step, in Tables 2, 3, and 4 below.

4.2 Evaluation of Component and Stage Reliabilities Considering Cost, Load, and Size Constraints Through the LMT Without Rounding in Series-Parallel Systems

Figures 2 and 3 display the performance outcomes of components and various process stages under the limitations of price, load, and volume. These results were generated after approximately 40 cycles of experimentation using a MATLAB-based trial-and-error technique. The software was specifically implemented to formulate a comprehensive redundant reliability framework within a series-parallel arrangement, incorporating factors like budget, capacity, and dimensional restrictions.

Phase of the Process (R_{dep}) and Component (r_{dep}) Time saved Obtained Using LMT Without Applying Rounding-Off





4.3 Comprehensive Assessment of Component Cost Limitations Utilizing the LMT Without Rounding in Series-Parallel Configuration Systems

Table 2 Evaluation of Cost Limitations in Series-Parallel Network Structures Through the Lagrange Multiplier Technique

Level	l_y	m_y	r_{dep}	Log r_{dep}	R_{dep}	*Log* R_{dep}	C_{com}	P_{cr}	$P_{cr} \cdot C_{com}$
I	5,000	0.91	0.9355	-0.0290	0.8464	-0.0724	2.50	3,679	9,201
II	20,000	0.95	0.9611	-0.0172	0.8925	-0.0494	2.87	14,912	42,744
III	40,000	0.98	0.9736	-0.0116	0.9001	-0.0457	3.93	30,020	1,18,096
Ultimate Value									1,70,041
Efficiency of System ($R_{sys\ dep}$)									0.9546

4.4 Comprehensive Assessment of Component Load Cost Limitations Utilizing the LMT Without Rounding in Series-Parallel Configuration Systems

Table 3: Evaluation of Load Limitations in Series-Parallel Network Structures Through the Lagrange Multiplier Technique

Level	p_y	q_y	r_{dep}	Log r_{dep}	R_{dep}	*Log* R_{dep}	C_{com}	W_{cr}	$W_{cr} \cdot C_{com}$
I	500	0.93	0.9355	-0.0290	0.8464	-0.0724	2.50	368	919.39
II	5,000	0.95	0.9611	-0.0172	0.8825	-0.0543	3.15	3,728	11,744.81
III	10,000	0.98	0.9736	-0.0116	0.9001	-0.0457	3.93	7,505	29,524.01
Ultimate Load									42,188.21
Efficiency of System ($R_{sys\ dep}$)									0.9546

4.5 Comprehensive Assessment of Component Size Limitations Utilizing the LMT Without Rounding in Series-Parallel Configuration Systems

Table 4: Evaluation of Size Limitations in Series-Parallel Network Structures Through the Lagrange Multiplier Technique

Level	u_y	v_y	r_{dep}	Log r_{dep}	R_{dep}	*Log* R_{dep}	C_{com}	V_{cr}	$V_{cr} \cdot C_{com}$
I	2000	0.92	0.9355	-0.0290	0.8464	-0.0724	2.50	1,471	3,679.0
II	5,000	0.95	0.9611	-0.0172	0.8925	-0.0494	2.87	3,728	10,686.1
III	10,000	0.97	0.9736	-0.0116	0.9001	-0.0457	3.93	7,506	29,528.5
Ultimate Dimension									43,893.55
Efficiency of System ($R_{sys\ dep}$)									0.9546

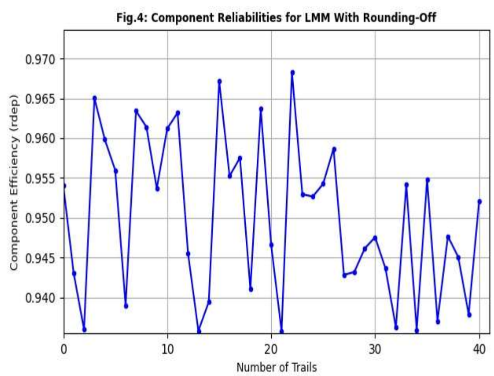
5. LAGRANGE MULTIPLIER-BASED OPTIMIZATION OF SYSTEM EFFICIENCY

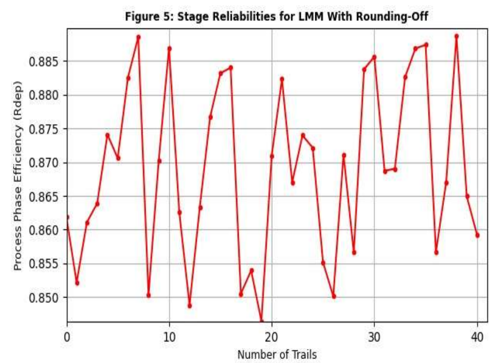
The system's efficiency incorporates ′pq′ values rounded ′pq′ to the nearest whole number, treating them as integers. The tables offer a full explanation of the acceptable outcomes for cost, load, and size. To collect the required information, you must determine the difference between the values before and after rounding ′pq′ to the nearest integer, as well as the variations caused by load, dimensions, value, and construction capacity.

5.1 Comprehensive Evaluation of Component Constraints—Price, Load, and Size—Using the Lagrange Multiplier Method with Rounding in Series-Parallel Configuration Systems

Results from around 40 iterations of a MATLAB-based trial-and-error method are shown in Figures 4 and 5, respectively, for the component and process phase efficiencies linked to size, cost, and load limitations. The goal of this effort was to include constraints like weight, size, and budget into the development of a unified redundant reliability model operating in a series-parallel framework.

Phase of the Process (R_{dep}) and Component (r_{dep}) Time saved Obtained Using LMT With Applying Rounding-Off





5.2 Analysis of Efficiency Design Using the LMT with Consideration of Value, Load, and Size Parameters, Incorporating Rounding-Off Techniques in Series-Parallel Configuration Systems

Table 5: *The following table presents the results of an analysis of the efficiency design based on load, size, value constraints, and the LMT with rounding-off techniques.*

Level	r_{dep}	R_{dep}	C_{com}	P_{cr}	$P_{cr} . C_{com}$	R_{dep}	W_{cr}	$W_{cr} . C_{com}$	R_{dep}	V_{cr}	$V_{cr} . C_{com}$
I	0.9355	0.8464	3	3,679	11,036	0.8464	369	922	0.8464	1,471	4408
II	0.9611	0.8825	3	14,912	44,737	0.8825	3,732	11,758	0.8925	3,728	11193
III	0.9736	0.8895	4	30,020	1,20,082	0.8901	7,507	32,668	0.9001	7,506	30028
Total Worth, Load and Size				1,75,855		45,347			45,628		
Efficiency of System ($R_{sys\ dep}$)									0.9674		

5.2.1 Examining the Impact of LMT on the Price-Component Variation = 03.42%
5.2.2 Examining the Impact of LMT on the Load-Component Variation = 07.48%
5.2.3 Examining the Impact of LMT on the Size-Component Variation = 03.95%
5.2.4 Variations in System Efficiency with HAA = 01.34%

6. HEURISTIC LOGIC IN RELIABILITY OPTIMIZATION:

Finding a predetermined set of rules or processes that work well in a given context is what heuristic problem-solving is all about. Instead of using an optimization method, a heuristic approach is usually employed when the problem cannot be adequately classified using normal methods. Even when the issue doesn't conform to a recognized category, factors like resource limitations, such as computational time or data requirements, may render optimization strategies impractical. The following list outlines characteristics of problems that may indicate the need for heuristics. Problems without proven algorithmic solutions are referred to as ill-structured.

6.1 Common Heuristic Techniques:

1. **Greedy Algorithm**: Makes the locally optimal choice at each stage with the hope of finding a global optimum.
2. **Simulated Annealing**: Mimics the process of heating and slowly cooling material to find a minimum energy state, exploring the solution space probabilistically.
3. **Genetic Algorithms**: Inspired by natural selection, these algorithms evolve solutions through crossover, mutation, and selection.
4. **Tabu Search**: Uses local search methods to find a solution, while avoiding revisiting previously explored solutions.
5. **Ant Colony Optimization**: Inspired by the behavior of ants searching for food, this algorithm uses pheromone trails to find good paths in complex problems.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

In the current IRR model, the author employed the Tabu Search method to analyze key factors such as the number of components, component reliabilities, stage reliabilities, and overall system reliabilities. This technique was used to explore and optimize the system's performance while avoiding previously explored solutions, helping to find a near-optimal configuration for the system.

6.2 Heuristic Algorithm

In computer science, a Heuristic Algorithm (or simply Heuristic) is a method used to find an acceptable solution to a problem in various real-world situations. While it mimics the general approach of heuristics, it lacks formal proof of its correctness. Although a heuristic may provide an accurate solution, it is not guaranteed to be optimal or resource-efficient. Heuristics are commonly applied when there is no known approach for determining an optimal solution within the given constraints or for the problem in general.

In the work proposed by the author, novel heuristic processes were developed to improve the system's reliability in series-parallel configuration redundant reliability systems with various constraints. The methodology for the heuristic algorithm is described below.

6.3 Heuristic Solution Approach

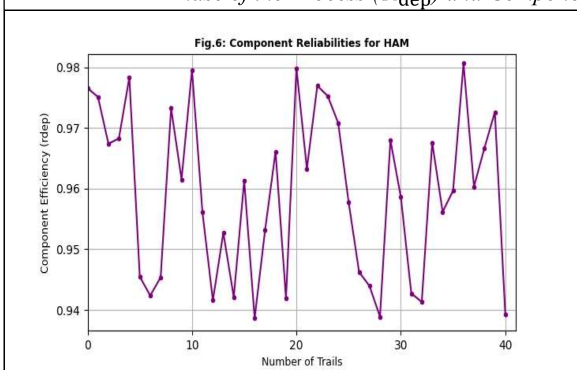
Utilizing the heuristic method for the suggested mathematical function, integrated reliability models for series-parallel redundant systems are created and fine-tuned while dealing with various limitations. Taking into account the case problem mentioned earlier, we want to produce an optimum design. The heuristic method is fed the component reliabilities and the number of components in each stage.

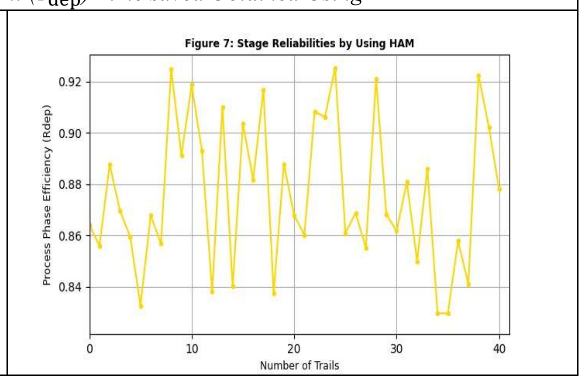
The approach excels in optimizing designs with integer component values, which makes it very applicable to real-world scenarios. Using the heuristic technique, we determine and analyze and offer the results for the mathematical function under investigation. These results include numerous weight, volume, and cost-related quantities.

6.4 Improving Process Phase Efficacy and Component Reliability with the Use of Heuristic Algorithms

The component and process phase efficiencies (r_{dep} and R_{dep} respectively) under the size constraint are shown in Figures 6 and 7, respectively. These efficiencies were obtained using around 40 iterations of a trial-and-error procedure utilizing the Heuristic Algorithm Approach in the MATLAB software. Taking size, weight, and cost into account, this program's designers built an integrated redundant reliability model for use in a series-parallel architecture.

Phase of the Process (R_{dep}) and Component (r_{dep}) Time saved Obtained Using HAA





6.5 Thorough Assessment of Series-Parallel Configuration Systems' Component Cost Constraints through the Use of Heuristic Algorithms

The authors determined the ideal component (r_{dep}) and process phase (R_{dep}) efficiencies according to price limitations generated from the iterative approach; Table 6 displays the value-related efficiency design.

Table 6 Evaluation of Cost Limitations in Series-Parallel Configuration Systems Through the Heuristic Algorithm Approach

Level	l_y	m_y	r_{dep}	Log r_{dep}	R_{dep}	$Log\ R_{dep}$	C_{com}	P_{cr}	$P_{cr} . C_{com}$
I	5,000	0.91	0.9366	-0.0284	0.8215	-0.0854	3	3,681	11,050
II	20,000	0.95	0.9601	-0.0177	0.8851	-0.0530	3	14,904	44,675
III	40,000	0.98	0.9761	-0.0105	0.9077	-0.0421	4	30,063	1,20,352
Ultimate Value									1,76,078
Efficiency of System ($R_{sys\ dep}$)									0.9799

6.6 Thorough Assessment of Series-Parallel Configuration Systems' Component Load Constraints through the Use of Heuristic Algorithms

The authors determined the ideal component (r_{dep}) and process phase (R_{dep}) efficiencies according to load limitations generated from the iterative approach; Table 7 displays the load-related efficiency design.

Table 7: Evaluation of Load Limitations in Series-Parallel Configuration Systems Through the Heuristic Algorithm Approach

Level	p_y	q_y	r_{dep}	Log r_{dep}	R_{dep}	$Log\ R_{dep}$	C_{com}	W_{cr}	$W_{cr} . C_{com}$
I	500	0.93	0.9435	-0.0253	0.8398	-0.0758	3	369	1,109
II	5,000	0.95	0.9704	-0.0130	0.9137	-0.0392	3	3,748	11,257
III	10,000	0.98	0.9822	-0.0078	0.9306	-0.0312	4	7,542	30,202
Ultimate Load									42,568
Efficiency of System ($R_{sys\ dep}$)									0.9799

6.7 Thorough Assessment of Series-Parallel Configuration Systems' Component Size Constraints through the Use of Heuristic Algorithms

Based on the size constraints imposed by the iterative method, the authors calculated the optimal component (r_{dep}) and process phase (R_{dep}) efficiencies; the design of these efficiencies is shown in Table 8.

Table 8: Evaluation of Size Limitations in Series-Parallel Configuration Systems Through Heuristic Algorithm Approach

Level	u_y	v_y	r_{dep}	Log r_{dep}	R_{dep}	$Log\ R_{dep}$	C_{com}	V_{cr}	$V_{cr} . C_{com}$
I	2000	0.92	0.9501	-0.0222	0.8575	-0.0668	3	1,483	4,454
II	5,000	0.95	0.9622	-0.0167	0.8909	-0.0502	3	3,730	11,184
III	10,000	0.97	0.9785	-0.0094	0.9168	-0.0377	4	7,527	30,083
Ultimate Dimension									45,721
Efficiency of System ($R_{sys\ dep}$)									0.9799

6.7.1 Examining the Impact of HAA on the Price-Component Variation = 00.12%
6.7.2 Examining the Impact of HAA on the Load-Component Variation = 06.13%
6.7.3 Examining the Impact of HAA on the Size-Component Variation = 04.73%
6.7.4 Variations in System Efficiency with HAA = 01.29%

Integrating heuristics into the suggested mathematical function allows for the creation of reliability models for redundant systems subject to many constraints. The heuristic strategy generates inputs for the present case problem using the Lagrangean method. The heuristic technique generates integer numbers for the number of components in each step (C_{com}), making it ideal for real-world applications. Though they can produce integer solutions, the computation time of traditional methods such as integer or dynamic programming increases exponentially with the complexity of the problem. The heuristic technique easily finds integer solutions, making complex systems feasible and scalable.

While the heuristic approach does provide a solution, it is only a rough approximation, which is its biggest flaw. It might not necessarily give a limited or closed answer, but it's realistic and possible nonetheless. If you're dealing with an industrial problem with a lot of variables, this strategy will help you get near-optimal solutions.

The author then set out to solve the provided mathematical models exactly using integer values by employing the integer programming technique. The results of this approach are detailed in this section of the paper.

7. INTEGER PROGRAMMING TECHNIQUE:

It may be challenging to apply the Lagrangian method since it needs precise values to specify the quantity of components ('pq') at each step.

Truncating result values commonly changes characteristics like cost, load, and size, affecting system reliability and model efficiency. The author presents a integer programming-based empirical strategy to find integer solutions. This method uses Lagrangian method outputs as integer programming input parameters, providing a more practical and accurate solution.

7.1 Improving System Dependability and Efficient Use of Resources in a Series-Parallel Architecture through the Application of Integer Programming

By taking the component reliabilities into account, integer programming can calculate the total system dependability, the number of components in each stage, and the reliabilities of the individual stages. While integer programming is great for building integrated reliability models, it has a major drawback in that it cannot be utilized directly without entering the component reliabilities. Integer programming provides a solution to this problem by taking the component reliabilities from the previous approach, the Lagrangian method, and producing the stage reliabilities, system reliabilities, and component numbers for each stage as output.

Integer programming offers flexibility in selecting the number of components at each step, the reliability of each stage, and the overall system reliability, all while adhering to the constraints defined by the problem. This enables a more tailored and optimized solution within the provided limits.

7.2 Integer Programming Method

Integer Programming (IP) solves optimization problems with integer decision variables. This method is appropriate for discrete decisions like system component or stage counts. General steps for tackling integer programming problems: Step 1: Define decision variables that indicate problem decisions. The number of components and stages will be integers.

Step 2: Create the objective function and set the optimization goal, such as system dependability or cost reduction. A mathematical statement involving decision variables is the objective function.

Step 3: Establish Limits and define decision variable constraints. These constraints may limit resources, component capacity, system reliability, etc. Most constraints are linear, but complex issues may include nonlinear constraints.

Step 4: Enter Component Reliabilities or Other parameters provide values from past methods (e.g., the Lagrangian method) if the problem concerns dependability or other parameters. These values will feed the Integer Programming model.

Step 5: Solve the integer programming problem Find the best solution using branch-and-bound, cutting planes, or simplex approaches. The solver will choose choice variables (e.g., number of components, stage reliabilities) that maximize or reduce the objective function while fulfilling all constraints.

Step 6: Interpret Solution after optimisation, analyse the outcomes. This includes interpreting component counts, stage reliabilities, and system performance. Based on objectives and restrictions, the solution should deliver the best configuration.

7.3 Results of Integer Programming Method

We used the Lagrange multiplier approach to continuously solve the series-parallel Integrated Redundant Reliability (IRR) systems that were recommended. To enhance system reliability, reliability engineers use these topologies, which integrate components in series and parallel. For use in the tested models, the method yielded integer and real-valued (continuous) solutions.

The Integer Programming Approach was used to study and comprehend these models' major results. This method optimized judgments at each stage, ensuring optimal management of series and parallel components in the IRR system to maximize reliability. Tables 9, 10 and 11 shows the mathematical function's performance and reliability results. This table shows how the Lagrange multiplier, Heuristic Algorithm, and Integer Programming methods solved the IRR model's series-parallel reliability problems.

7.4 Complete IP Evaluation of Component Cost Constraints in Series-Parallel Configuration Systems

Table 9: *Analyzing Series-Parallel Configuration Systems' Component Cost Constraints in Depth with the IPM*

Level	l_y	m_y	r_{dep}	Log r_{dep}	R_{dep}	Log R_{dep}	C_{com}	P_{cr}	$P_{cr} . C_{com}$
I	5,000	0.91	0.9499	-0.0223	0.8571	-0.0670	3	3,709	11,126
II	20,000	0.95	0.9724	-0.0122	0.9194	-0.0365	3	15,007	45,059
III	40,000	0.98	0.9894	-0.0046	0.9583	-0.0185	4	30,288	1,21,061
Ultimate Value									1,77,246
Efficiency of System ($R_{sys\ dep}$)									0.9823

7.5 Complete IP Evaluation of Component Load Constraints in Series-Parallel Configuration Systems

Table 10: *Analyzing Series-Parallel Configuration Systems' Component Load Constraints in Depth with the IPM*

Level	p_y	q_y	r_{dep}	Log r_{dep}	R_{dep}	Log R_{dep}	C_{com}	W_{cr}	$W_{cr} . C_{com}$
I	500	0.93	0.9601	-0.0177	0.8849	-0.0531	3	373	1,119
II	5,000	0.95	0.9704	-0.0130	0.9137	-0.0392	3	3,748	11,257
III	10,000	0.98	0.9822	-0.0078	0.9307	-0.0312	4	7,542	30,157
Ultimate Load									42,533
Efficiency of System ($R_{sys\ dep}$)									0.9823

7.6 Complete IP Evaluation of Component Size Constraints in Series-Parallel Configuration Systems

Table 11: *Analyzing Series-Parallel Configuration Systems' Component Size Constraints in Depth with the IPM*

Level	u_y	v_y	r_{dep}	Log r_{dep}	R_{dep}	Log R_{dep}	C_{com}	V_{cr}	$V_{cr} . C_{com}$
I	2000	0.92	0.9655	-0.0152	0.8999	-0.0458	3	1,496	4,494
II	5,000	0.95	0.9786	-0.0094	0.9372	-0.0282	3	3,765	11,287
III	10,000	0.97	0.9894	-0.0046	0.9583	-0.0185	4	7,572	30,267
Ultimate Size									46,048
Efficiency of System ($R_{sys\ dep}$)									0.9823

7.6.1 Examining the Impact of IPM on the Price-Component Variation = 00.12%
7.6.2 Examining the Impact of IPM on the Load-Component Variation = 06.13%
7.6.3 Examining the Impact of IPM on the Size-Component Variation = 04.73%
7.6.4 Variations in System Efficiency with IPM = 01.29%

Journal of Theoretical and Applied Information Technology
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ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

8. ANALYTICAL COMPARISON:

The results of four different approaches are compared in this study: integer programming, rounding-off, heuristic technique, and Lagrangean multiplier. The study looks at the whole system reliability, component reliability, stage reliability, and the number of components.

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8.1 Improving Series-Parallel Configuration Systems' IRR about Cost through the Use of LMT with Rounding, HAM, and IPM

Table 12: *The LMT's Relationship with Rounding Methods, HAM, and IPM as They Relate to Series-Parallel Hardware Pricing*

Maximum-Cost		Assembling a Final			Heuristic Algorithm			Integer Programming		
Level	C_{com}	r_{dep}	R_{dep}	$P_{cr} . C_{com}$	$r_{Com\ Re}$	R_{dep}	$P_{cr} . C_{com}$	$r_{Com\ Re}$	R_{dep}	$P_{cr} . C_{com}$
I	3	0.9355	0.8464	11,036	0.9366	0.8215	11,050	0.9499	0.8571	11,126
II	3	0.9611	0.8825	44,737	0.9601	0.8851	44,675	0.9724	0.9194	45,059
III	4	0.9736	0.8895	1,20,082	0.9761	0.9077	1,20,352	0.9894	0.9583	1,21,061
Efficiency of System ($R_{sys\ dep}$)		1,75,855			1,76,078			1,77,246		
		LMT SE = 0.9674			HAA SE = 0.9799			IPM SE = 0.9823		

8.2 Improving Series-Parallel Configuration Systems' IRR about Load through the Use of LMT with Rounding, HAM, and IPM

Table 13: *The LMT's Relationship with Rounding Methods, HAM, and IPM as They Relate to Series-Parallel Hardware Loading*

Maximum-Mass		Assembling a Final			Heuristic Algorithm			Integer Programming		
Level	C_{com}	r_{dep}	R_{dep}	$W_{cr} . C_{com}$	r_{dep}	R_{dep}	$W_{cr} . C_{com}$	r_{dep}	R_{dep}	$W_{cr} . C_{com}$
I	3	0.9355	0.8464	922	0.9435	0.8398	1,109	0.9601	0.8849	1,119
II	3	0.9611	0.8825	11,758	0.9704	0.9137	11,257	0.9704	0.9137	11,257
III	4	0.9736	0.8901	32,668	0.9822	0.9306	30,202	0.9822	0.9307	30,157
Efficiency of System ($R_{sys\ dep}$)		45,347			42,568			42,533		
		LMT SE = 0.9674			HAA SE = 0.9799			IPE SE = 0.9823		

8.3 Improving Series-Parallel Configuration Systems' IRR about Size through the Use of LMT with Rounding, HAM, and IPM

Table 14: *The LMT's Relationship with Rounding Methods, HAM, and IPM as They Relate to Series-Parallel Hardware Sizing*

Maximum-Size		Assembling a Final			Heuristic Algorithm			Integer Programming		
Level	C_{com}	r_{dep}	R_{dep}	$V_{cr} . C_{com}$	r_{dep}	R_{dep}	$V_{cr} . C_{com}$	r_{dep}	R_{dep}	$V_{cr} . C_{com}$
I	3	0.9355	0.8464	4408	0.9501	0.8575	4,454	0.9655	0.8999	4,494
II	3	0.9611	0.8925	11193	0.9622	0.8909	11,184	0.9786	0.9372	11,287
III	4	0.9736	0.9001	30028	0.9785	0.9168	30,083	0.9894	0.9583	30,267
Efficiency of System ($R_{sys\ dep}$)		45,628			45,721			46,048		
		LMT SE = 0.9674			HAA SE = 0.9799			IPM = 0.9823		

9. CONCLUSION:

In order to determine the required number of components, as well as the reliabilities of individual components and stages, this study presents an enhanced integrated redundant reliability model that employs a series-parallel architecture. The goal of this model is to improve system reliability. Focusing on a typical Automation Forum setup involving interdependent units like compressors, vaporizers, and condensers arranged in parallel for backup, the study evaluates performance through three operational stages using the Lagrange multiplier method. While the method yielded real-valued efficiencies, these were impractical for real-world application, necessitating adjustments to express them as integers. This refinement improved both component and stage reliability while accounting for cost, weight, and volume. The study offers valuable insights into optimizing complex system reliability under realistic constraints.

This research presents a comprehensive reliability model that incorporates several efficiency criteria and is tailored to a system with a series-parallel structure. Essential metrics such as the number of components (C_{com}), their relative efficiency (r_{dep}), stage reliabilities (R_{dep}), and overall system reliability ($R_{sys\ dep}$) are calculated using the Lagrange multiplier approach once the data is validated to be real-valued. The efficiencies for price, load, and size derived from this analysis are quantified as r_{dep} = 0.9355, 0.9611, and 0.9736, with corresponding stage reliabilities of R_{dep} = 0.8464, 0.8825, and 0.8835. The calculated reliability of the resulting structure is $R_{sys\ dep}$ = 0.9524. Using Lagrangean multiplier techniques, the rounding-off efficiencies are quantified as r_{dep} = 0.9355, 0.9611, and 0.9736, with corresponding stage reliabilities of R_{dep} = 0.8464, 0.8925, and 0.9001. The calculated reliability of the resulting structure is $R_{sys\ dep}$ = 0.9674.

Because the previously given results produce scientifically incorrect rounded values, the author offered an alternate method, the Heuristic Algorithm, to get integer solutions. In order to obtain integer solutions, a heuristic procedure is used, with inputs taken from the Lagrange multiplier analysis for practical purposes. This yields revised component reliabilities for price of r_{dep} = 0.9366, 0.9601, and 0.9761, revised component reliabilities for load of r_{dep} = 0.9435, 0.9704, 0.9822, revised component reliabilities for size of r_{dep} = 0.9501, 0.9622, 0.9785 respectively, and the stage reliabilities of price is R_{dep} = 0.8215, 0.8851, and 0.9077, the stage reliabilities of load is R_{dep} = 0.8398, 0.9137, and 0.9306, and the stage reliabilities of size is R_{dep} = 0.8575, 0.8909, and 0.9168. The improved system reliability is quantified at $R_{sys\ dep}$ = 0.9799.

For real-world applications, we use inputs from the Lagrange multiplier analysis in an integer programming approach to get integer solutions. This yields revised component reliabilities for price of r_{dep} = 0.9499, 0.9724, and 0.9894, revised component reliabilities for load of r_{dep} = 0.9601, 0.9704, 0.9822, and revised component reliabilities for size of r_{dep} = 0.9655, 0.9786, 0.9894 respectively, and the stage reliabilities of price is R_{dep} = 0.8571, 0.9194, and 0.9583, the stage reliabilities of load is R_{dep} = 0.8849, 0.9137, and 0.9307, and the stage reliabilities of size is R_{dep} = 0.8999, 0.9372, and 0.9583. The improved system reliability is quantified at $R_{sys\ dep}$ = 0.9823. It is noteworthy that even though the changes made to the cost, weight, and dimensions of the components were relatively minor, these adjustments led to significant improvements in stage reliability, thereby enhancing the overall system reliability at each stage and for each component. The author came to the conclusion that, when compared to the Lagrange multiplier method with rounding and heuristic algorithm approaches, the integer programming method yields more accurate integer solutions.

This methodology produces a valuable reliability model, especially for series-parallel configurations with reliability redundancy. Because it helps design engineers choose materials that balance performance and cost, it benefits system dependability engineers, especially in scenarios with limited system value. A unique approach that sets minimum and maximum component reliability restrictions while optimizing system reliability is suggested for future research. Modern heuristic methods can generate similar integrated reliability models with redundancy, making them more useful in reliability engineering.

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