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PERFORMANCE ASSESSMENT OF NHPP SOFTWARE RELIABILITY MODELS APPLYING SHAPE PARAMETER OF WEIBULL-TYPE LIFE DISTRIBUTIONS

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ABSTRACT

This study was conducted with the aim of evaluating the performance properties issue of NHPP-based software reliability models parameterized by the shape parameter of the Weibull-type life distributions. For this purpose, failure time data collected from actual software fault occurrences were utilized, and the model parameters were estimated using the MLE approach. The reliability performance of the proposed models was then assessed through several analytical approaches, including model efficiency evaluation using MSE and R^2, the degree of accuracy in prediction against observed values using m(t), instantaneous failure rate analysis by $\lambda(t)$, and reliability prediction based on future mission times. The results revealed that the Inverse-Exponential model demonstrated superior efficiency compared with the other models. Accordingly, this study provides a novel analysis of performance issues in reliability models governed by the shape parameter of the Weibull distribution, an area insufficiently addressed in prior studies, and offers basic reliability properties that can be helpful to developers in the preparation stage.
Keywords: *Exponential, Inverse-Exponential, NHPP, Rayleigh, Property Assessment.*

1. INTRODUCTION

The era of the Fourth Industrial Revolution, driven by AI-based convergence technology, is accelerating the digital transformation of related industries through collaboration with advanced information technologies. Since these AI systems operate on large-scale software, ensuring software reliability is directly tied to system safety. Unlike conventional software, AI software exhibits highly complex and nonlinear structures, making the prediction of failure occurrence times as well as failure forecasting considerably difficult. Consequently, reliability models utilizing the Non-Homogeneous Poisson Process (NHPP) have attracted significant attention, as they enable quantitative modeling of the dynamic nature of failure rates that vary over time. Specifically, NHPP-based models can predict fault occurrence rates and quantitatively estimate the remaining number of failures using failure-time data collected during the software testing or operational phases. Moreover, NHPP reliability models can be flexibly combined with various lifetime distributions, allowing their application to diverse software failure characteristics. To address the complex operational environments and heterogeneous failure patterns of AI-convergent software systems, comprehensive performance and goodness-of-fit evaluations of different NHPP models are indispensable. However, most existing studies remain limited to a narrow set of distributions, and related NHPP reliability models have been investigated primarily within this restricted scope [1]. In this regard, Yang [2] analyzed a finite-failure software reliability model incorporating the properties of the Weibull lifetime distribution by employing the NHPP. Furthermore, Song, Chang, and Pham [3] proposed a Weibull-based fault-detection rate within an NHPP framework to account for environmental variations, while Kim [4] evaluated the property of NHPP software reliability models by comparing the Makeham and Extended-Weibull distributions. Similarly, Hanagal and Bhalerao [5] employed an NHPP model using the maximum likelihood estimation (MLE) approach to propose various extended forms of the inverse Weibull distribution. Along the same line, Wang, Geng, and Li [6] introduced a Weibull–Weibull NHPP reliability model for big-data systems, grounded in predictive

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accuracy analysis. Finally, Yang and Park [7] applied the Weibull-Extension and Flexible Weibull-Extension distributions, both derived from the Weibull distribution, to NHPP reliability models and presented a novel approach for analyzing software reliability performance.

Accordingly, this study applies various life distributions determined by changes in the shape parameter of Weibull-type distributions to NHPP software reliability models and analyzes the property that governs their performance. Consequently, the efficiency and performance of the resulting models are comparatively evaluated, and the optimal model is proposed based on the derived results.

2. RELATED RESEARCH

2.1.1 NHPP model

The NHPP model, formulated as a stochastic process with time-varying failure rates, is widely utilized to characterize time-dependent event occurrences such as software failures. Accordingly, to examine the probabilistic nature of irregular failure occurrences in software systems, the reliability performance of the NHPP model is assessed using actual failure times recorded during system operation. For the purpose of modeling, the cumulative number of failures is expressed as N(t), the average expected failures as m(t), and the rate of failure occurrence as λ(t). Based on this formulation, N(t) is characterized by a Poisson probability distribution with parameter m(t), as illustrated by the following expression.

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \tag{1}$$

Accordingly, with respect to the performance of the model, m(t) is used to denote the anticipated failure counts over time, while λ(t) characterizes the immediate failure intensity at any given moment. These functional relationships are mathematically specified by the following expression.

$$m(t) = \int_0^t \lambda(s)ds \tag{2}$$

$$\frac{dm(t)}{dt} = \lambda(t) \tag{3}$$

2.1.2 NHPP software reliability model

The reliability model based on the NHPP has been extensively employed to statistically analyze and predict the failure behavior of software systems over time. This model is especially appropriate for describing cases in which defect occurrences vary over time, thereby accommodating irregular failure intervals. Consequently, developers can apply this model to identify potential defects, estimate the required testing duration and resources, and assess the level of software reliability prior to deployment.

In this work, the finite failure condition is applied, which assumes a realistic failure environment in which no additional defects arise during the removal process. Under this assumption, the number of detectable residual defects at testing time t is denoted by θ. Also, when utilizing F(t), denoting the probability of failure up to time t, and f(t), which expresses the momentary failure intensity around time t, the property functions representing model performance can be derived as shown by the following expression.

$$\mathrm{m(t)} = \theta F(\mathrm{t}) \tag{4}$$

$$\lambda(\mathrm{t}) = \frac{dm(t)}{dt} = \theta f(t) \tag{5}$$

Thus, the likelihood function for the model based on NHPP is given by the following expression [8].

$$L_{NHPP}(\Theta|\underline{x}) = \left(\prod_{i=1}^{n} \lambda(x_i)\right) exp[-m(x_n)] \tag{6}$$

2.2 NHPP Exponential-Basic Model

The exponential distribution is a particular form of the Weibull distribution and represents the most fundamental lifetime distribution with a constant failure rate. Accordingly, the Exponential-Basic model, in which the lifetime distribution follows the basic exponential form, is also referred to as the Goel–Okumoto basic model. Moreover, due to its exponential-type characteristics, it is regarded as a representative baseline model in the domain of reliability. Accordingly, considering the shape parameter (α), the Weibull distribution function is given by the following relationship.

$$F(t) = 1 - e^{-bt^{\alpha}} \tag{7}$$

$$f(t) = \alpha b t^{\alpha-1} e^{-bt^{\alpha}} \tag{8}$$

When the shape parameter of this distribution is α=1, as specified in Equations (7) and (8), the distribution becomes an Exponential-Basic distribution. If the remaining faults are denoted by θ, the attribute functions that determine reliability performance are given in Equations (9) and (10) [9].

$$m(t) = \theta(1 - e^{-bt}) \tag{9}$$

$$\lambda(t) = \theta b e^{-b} \tag{10}$$

Subsequently, by substituting into Equation (6) and applying the logarithmic transformation to both sides, the log-transform function is obtained as expressed in Equation (11).

$$lnL_{NHPP}(\Theta|\underline{x}) = nln\theta + nlnb - b\sum_{k=1}^{n} x_k - \theta(1 - e^{-bx_n}) \tag{11}$$

To estimate the parameters, the log-likelihood function in Equation (11) is partially differentiated by θ and b, yielding the results shown in Equations (12) and (13). Finally, by applying the bisection method to these equations, the maximum likelihood estimators of this model's parameters, $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$, can be determined.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \tag{12}$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta}x_n e^{-\hat{b}x_n} = 0 \tag{13}$$

2.3 NHPP Inverse-Exponential Model

The Weibull distribution is generally regarded as appropriate for characterizing rising failure rates, whereas its inverse form, the Inverse-Weibull distribution, is particularly effective in describing declining failure rate patterns. In particular, owing to this inverse-exponential property, the Inverse-Weibull distribution has been widely applied as a lifetime distribution to describe failure intervals in NHPP-based software reliability models.

As given in Equation (14), the Inverse-Weibull distribution is defined by the cumulative distribution function.

$$F(\mathrm{t}) = e^{-(bt)^{-\gamma}} \tag{14}$$

When the shape parameter of the Inverse-Weibull distribution is γ=1, as specified in Equation (14), the distribution becomes an Inverse-Exponential distribution. Accordingly, if the remaining faults are denoted by θ, the attribute functions that determine reliability performance are given in Equations (15) and (16) [10].

$$\mathrm{m(t)} = \theta e^{-(bt)^{-1}} \tag{15}$$

$$\lambda(\mathrm{t}) = \theta b^{-1}t^{-2}e^{-(bt)^{-1}} \tag{16}$$

Consequently, by substituting into Equation (6) to obtain the likelihood function and applying the logarithmic transformation, the log-transform function is obtained as expressed in Equation (17).

$$\mathrm{ln}L_{NHPP}(\Theta|\underline{x}) = nln\theta - nlnb + 2\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}(bx_i)^{-1} - \hat{\theta}e^{-(bx_n)^{-1}} = 0 \tag{17}$$

Estimation of θ and b is achieved by differentiating the log-likelihood function with respect to these parameters, which leads to Equations (18) and (19). Finally, by applying the bisection method to these equations, the maximum likelihood estimators of this model's parameters, $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$, can be determined as follows.

$$\frac{\partial \mathrm{ln}L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - e^{-(\hat{b}x_n)^{-1}} = 0 \tag{18}$$

$$\frac{\partial \mathrm{ln}L_{NHPP}(\Theta|\underline{x})}{\partial b} = -\frac{n}{\hat{b}} + \frac{1}{\hat{b}^2}\sum_{i=1}^{n}\frac{1}{x_i} - \theta\frac{1}{b^2x_n}e^{-(\hat{b}x_n)^{-1}} = 0 \tag{19}$$

2.4 NHPP Rayleigh Model

The Rayleigh distribution, which represents a special case of the Weibull lifetime distribution with exponential characteristics, is characterized by a low initial failure rate that increases linearly over time. Due to this attribute, it has been extensively applied as a fundamental lifetime distribution in reliability analysis. Accordingly, the Weibull distribution, taking the shape parameter (α) into account, is defined by the following expression.

$$F(t) = 1 - e^{-bt^{\alpha}} \tag{20}$$

$$f(t) = \alpha bt^{\alpha-1}e^{-bt^{\alpha}} \tag{21}$$

When the shape parameter of this distribution is α=2, as specified in Equations (20) and (21), the distribution becomes the Rayleigh distribution. If the remaining faults are denoted by θ, the attribute functions that determine reliability performance are given in Equations (22) and (23) [11].

$$m(t) = \theta\left(1 - e^{-bt^2}\right) \tag{22}$$

$$\lambda(t) = 2\theta bte^{-bt^2} \tag{23}$$

Subsequently, after substituting into Equation (6) and applying a logarithmic transformation to both sides, the final expression is obtained as shown in Equation (24).

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln2 + nln\theta + nlnb + \sum_{i=1}^{n} lnx_i$$
$$-b\sum_{i=1}^{n} x_i{}^2 - \theta\left(1 - e^{-bx_n^2}\right) \tag{24}$$

The maximum likelihood estimators ($\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$) may be computed using a numerical procedure such as the bisection method by the following expression.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + exp\left(-\hat{b}x_n^2\right) = 0 \tag{25}$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_i^2$$
$$-\hat{\theta}x_n^2\, exp\left(-\hat{b}x_n^2\right) = 0 \tag{26}$$

3. RELIABILITY PERFORMANCE ANALYSIS

Reliability performance is assessed in this work through the analysis of failure times collected from desktop computer usage. A total of 30 failure data points, which occurred randomly over 187.35 hours, were utilized for the analysis, and their summary is presented in Table 1 [12]. These failures are considered to have originated from design defects and testing errors introduced during the early stage of software development. The suitability of the collected data for reliability analysis was verified through the Laplace trend test, and the findings are depicted in Figure 1.

In general, a Laplace trend test outcome within 'the range of –2 to 2' denotes that the data contain no extreme observations, implying that the data are stable and thus appropriate for analysis.

Table 1: Software Failure Time.

Failure number	Failure time (hours)	Failure time (hours)$\times 10^{-2}$
1	4.79	0.0479
2	7.45	0.0745
3	10.22	0.1022
4	15.76	0.1576
5	26.10	0.261
6	35.59	0.3559
7	42.52	0.4252
8	48.49	0.4849
9	49.66	0.4966
10	51.36	0.5136
11	52.53	0.5253
12	65.27	0.6527
13	69.96	0.6996
14	81.70	0.817
15	88.63	0.8863
16	107.71	1.0771
17	109.06	1.0906
18	111.83	1.1183
19	117.79	1.1779
20	125.36	1.2536
21	129.73	1.2973
22	152.03	1.5203
23	156.40	1.564
24	159.80	1.598
25	163.85	1.6385
26	169.60	1.696
27	172.37	1.7237
28	176.00	1.76
29	181.22	1.8122
30	187.35	1.8735

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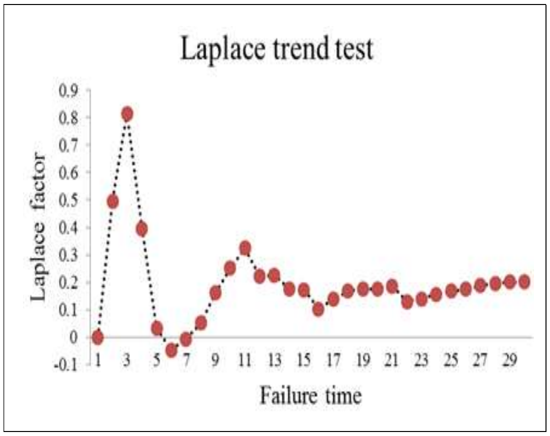

Figure 1: Analysis Results by Laplace Trend Test.

According to the trend test analysis shown in Figure 1, all values fall within the range of –2 to 2, indicating the absence of extreme values in the dataset. Consequently, the data on failure times, as listed in Table 1, are thus regarded as valid for conducting reliability analysis. The two parameters ($\hat{\theta}$ and $\hat{b}$) of the model were subsequently estimated through the MLE approach, and the outcomes are presented in Table 2 [13].

Table 2: Parameter Estimation by MLE.

Type	Model	MLE	
		$\hat{\theta}$	$\hat{b}$
Basic	Exponential	32.9261	0.1297
Weibull-type life distribution	Inverse-Exponential	41.2881	0.1692
	Rayleigh	30.0412	0.0188

Specifically, $\hat{\theta}$ indicates the remaining number of software failures, whereas $\hat{b}$ serves as the shape parameter characterizing the lifetime distribution pattern.

3.1. Efficient Model Identification

The model efficiency of the present study was evaluated using two key indicators. First, the R^2 property was employed to select an appropriate and efficient model based on predictive capability. Second, the MSE property was utilized to assess both accuracy and efficiency. R^2, referred to as the coefficient of determination, is a representative measure of the ability to explain a model and takes values between 0 and 1. Thus, R^2 is defined in Equation (27), and a larger R^2 value implies smaller error.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (27)$$

Note that $\hat{m}(x_i)$ corresponds to the accumulated failure counts up to time x_i, as derived from m(t).

Consequently, in the process of identifying an effective model, models with larger R^2 values are regarded as relatively more useful.

MSE, represented by the average of squared gaps between the model's predicted outcomes and the corresponding observations, serves as a key indicator of model performance and is as follows.

$$MSE = \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \quad (28)$$

Note that n stands for the recorded faults, whereas k indicates the parameter count.

Figure 2 depicts the MSE-based performance analysis outcomes of the proposed models. Accordingly, the MSE analysis results are employed as reference data for selecting an efficient model.

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When evaluating the proposed models, a lower MSE indicates superior predictive performance. MSE is derived by computing the squared deviations of the predicted values from the observed data, with smaller values corresponding to reduced prediction errors. Therefore, models with lower MSE values provide more accurate and reliable performance.

Table 3 presents a detailed comparison and analysis of MSE variations for each failure instance, serving as a reference for identifying the most efficient model.

Table 3: MSE Analysis.

Failure number	MSE		
	Exponential -Basic	Inverse-Exponential	Rayleigh
1	0.03453	0.03571	0.02707
2	0.03808	0.14074	0.10178
3	0.04224	0.29475	0.20844
4	0.15556	0.32769	0.24693
5	0.70898	0.01803	0.06889
6	1.36122	0.12164	0.00477
7	1.72884	0.38519	0.09800
8	1.94036	0.63108	0.26676
9	1.57239	0.45240	0.16438
10	1.29101	0.33523	0.10884
11	0.99079	0.20621	0.04797
12	1.65355	0.78708	0.74104
13	1.57358	0.80217	0.91838
14	2.01678	1.29812	1.99616
15	2.00657	1.37039	2.39014
16	2.75454	2.20186	4.04999
17	2.24224	1.75723	3.45139
18	1.85450	1.43509	3.00933
19	1.64185	1.28511	2.78372
20	1.48512	1.18805	2.56570
21	1.20368	0.95834	2.15716
22	1.43678	1.28115	2.09097
23	1.11811	1.00133	1.62184
24	0.81673	0.73077	1.19901
25	0.56976	0.51176	0.83943
26	0.38346	0.35203	0.54504
27	0.20666	0.18945	0.30629
28	0.08774	0.08156	0.13613
29	0.02213	0.02274	0.03420
30	0.00002	0.00049	0.00000

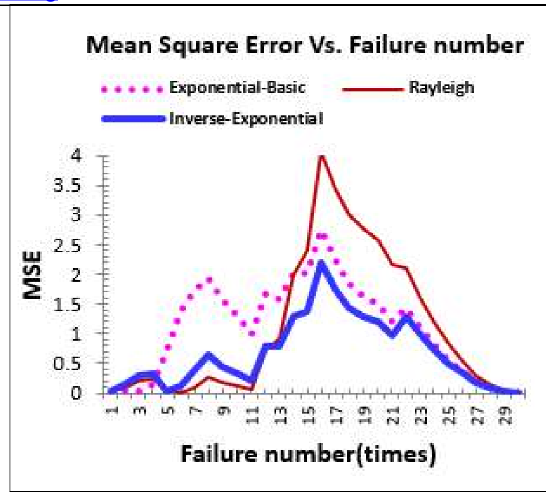

Figure 2: Property Analysis by MSE.

Table 4: Efficient Model Selection.

Type	Model	R^2	MSE
Basic	Exponential	0.8956	32.9379
Weibull-type life distribution	Inverse-Exponential	0.9359	20.2035
	Rayleigh	0.8980	32.1798

In general, a model is regarded as efficient when the R^2 value exceeds 0.8 (80%) and the MSE value is relatively low. Based on the data presented in Table 4, the analysis indicates that the Inverse-Exponential model demonstrates the highest efficiency and can be considered the most effective option for predictive performance [14].

3.2. Mean Value Function (m(t))

From the perspective of NHPP-based models, m(t) provides a critical indicator by expressing the predicted cumulative failure count that will occur by the test duration t. Put differently, this function serves as an indicator for forecasting how many failures are likely to occur as time progresses, and allows users to estimate how many failures may occur during software testing or operation.

In Table 5, the equations for determining m(t) in the proposed models are summarized and compared.

Figure 3 displays the changing pattern of the m(t) function, indicating its predictive accuracy for the true value. Analysis of the results indicates that the Inverse-Exponential model demonstrates the highest reliability [15].

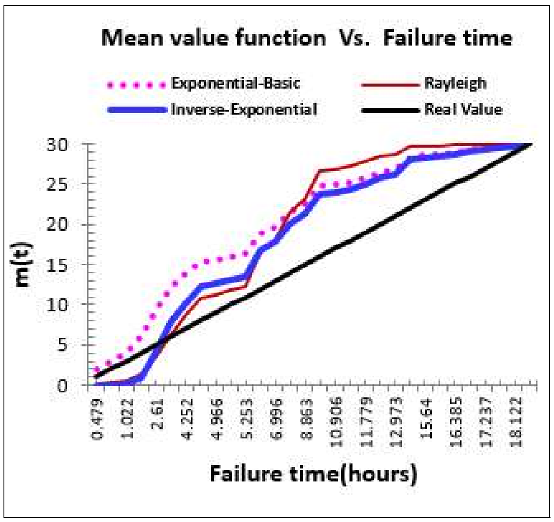

Figure 3: Performance Analysis by $m(t)$.

3.3. Intensity Function (λ(t))

λ(t) in NHPP-based reliability models characterizes the software failure hazard at any particular time. Combined with m(t), it is considered a fundamental performance indicator for reliability assessment, representing the instantaneous failure rate at time t.

Table 6: Intensity Function ($\lambda(t)$).

Type	Model	λ(t)
Basic	Exponential	$\theta b e^{-bt}$
Weibull-type life distribution	Inverse-Exponential	$\theta b^{-1} t^{-2} e^{-(bt)^{-1}}$
	Rayleigh	$2\theta b t e^{-bt^2}$

In Table 6, the equations for calculating λ(t) in the present study are summarized and compared.

Figure 4 presents the findings of the analysis conducted using the λ(t) function, which reflects the failure occurrence intensity over the failure duration.

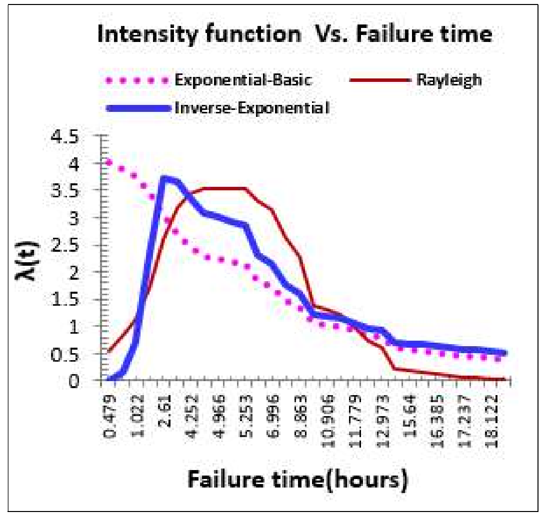

Figure 4: Performance Analysis by $\lambda(t)$.

Simulation results indicate that the Inverse-Exponential and Rayleigh models exhibit a failure rate pattern similar to physical failure mechanisms, initially low, rising substantially over time, and then gradually decreasing as faults are repaired. These findings suggest that the models derived in present study are effective with respect to model fit. Conversely, the Exponential-Basic model's intensity function indicates the highest early failure rate among the evaluated models, highlighting its relative

Table 5: Mean Value Function ($m(t)$).

Type	Model	m(t)
Basic	Exponential	$\theta(1 - e^{-bt})$
Weibull-type life distribution	Inverse-Exponential	$\theta e^{-(bt)^{-1}}$
	Rayleigh	$\theta\left(1 - e^{-bt^2}\right)$

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inefficiency in terms of model fit. This study aims to systematically characterize the failure patterns of the software system over time based on the analysis of λ(t), with the goal of supporting the establishment of design and testing procedures during the early stages. Accordingly, appropriate utilization of these two functions enables effective evaluation of the proposed model's reliability and helps reduce unnecessary software testing steps in the early development phase [16].

Table 7 presents a detailed comparison and analysis of failure outcomes at different time points, based on the key property functions (m(t) and λ(t)) that critically affect model performance. Accordingly, this study utilizes the analysis results from Table 7 to identify the optimal model.

Table 7: Performance Analysis by Attribute Functions (m(t) and λ(t)).

Failure Time (hours) $\times 10^{-2}$	Performance Attribute Functions					
	m(t)			λ(t)		
	Exponential -Basic	Inverse-Exponential	Rayleigh	Exponential -Basic	Inverse-Exponential	Rayleigh
0.0479	1.983330	0.000180	0.129301	4.01327	0.00465	0.53872
0.0745	3.032657	0.014801	0.311834	3.87717	0.15769	0.83277
0.1022	4.087570	0.127152	0.584144	3.74035	0.71948	1.13195
0.1576	6.087030	0.970901	1.370526	3.48102	2.31026	1.69895
0.261	9.45549	4.289380	3.611132	3.04413	3.72146	2.59374
0.3559	12.17366	7.845569	6.365719	2.69159	3.66073	3.16821
0.4252	13.95757	10.28412	8.656513	2.46021	3.36186	3.41888
0.4849	15.3708	12.20361	10.73301	2.27690	3.06749	3.52031
0.4966	15.63528	12.55913	11.14539	2.24261	3.00985	3.52825
0.5136	16.01235	13.06374	11.74571	2.19372	2.92696	3.53310
0.5253	16.26708	13.40290	12.15906	2.16067	2.87067	3.53195
0.6527	18.80438	16.69450	16.55513	1.83158	2.31603	3.30968
0.6996	19.63779	17.73920	18.07097	1.72349	2.14208	3.14876
0.817	21.51465	20.02889	21.47614	1.48006	1.77342	2.63111
0.8863	22.49559	21.19443	23.18070	1.35283	1.59463	2.28624
1.0771	24.78221	23.85188	26.64890	1.05626	1.21509	1.37383
1.0906	24.92357	24.01444	26.83052	1.03792	1.19327	1.31658
1.1183	25.20597	24.33897	27.17939	1.00130	1.15023	1.20333
1.1779	25.78026	24.99860	27.82860	0.92681	1.06487	0.97993
1.2536	26.44852	25.76762	28.47582	0.84014	0.96907	0.73784
1.2973	26.80545	26.18011	28.77178	0.79384	0.91936	0.61919
1.5203	28.34272	27.98935	29.65162	0.59446	0.71570	0.22269
1.564	28.59527	28.29503	29.73882	0.56170	0.68365	0.17781
1.598	28.78210	28.52345	29.79416	0.53747	0.66015	0.14843
1.6385	28.99416	28.78540	29.84811	0.50997	0.63369	0.11895
1.696	29.27673	29.13958	29.90655	0.47332	0.59873	0.08586
1.7237	29.40551	29.30322	29.92851	0.45661	0.58289	0.07303
1.76	29.56700	29.51118	29.95236	0.43561	0.56306	0.05879
1.8122	29.78729	29.79802	29.97863	0.40710	0.53625	0.04263
1.8735	30.02718	30.11770	30.00028	0.37598	0.50712	0.02882

3.4. Reliability Function ($\widehat{R}(\tau)$)

$\hat{R}(\tau)$ serves as a key indicator for assessing the reliability of software systems exhibiting time-dependent failure rates. Therefore, $\hat{R}(\tau)$ provides essential information for predicting the reliability of the model, this study utilizes arbitrarily assigned mission times within the proposed model to forecast future reliability performance.

The $\hat{R}(\tau)$ function represents the probability of a failure occurring at a specific test point t while simultaneously indicating the likelihood that no failure occurs within the interval of confidence over the entire mission time τ. In other words, this function aims to assess future reliability behavior by incorporating an arbitrarily specified mission time after the last observed failure time ($x_n = 187.35 \times 10^{-2}$). Based on this definition, $\hat{R}(\tau)$ is expressed in Equation (29) [17].

$$\widehat{R}(\tau|x_n) = exp[-\{m(x_n+\tau) - m(x_n)\}] \qquad (29)$$

As shown in Figure 5, the results of analyzing the change in reliability pattern according to the mission time (145H×10^{-1}) show that the reliability of the proposed models gradually decreases over time. While most of the proposed models exhibit reduced stability as mission time progresses, the Rayleigh model, which maintains consistently higher reliability, may be regarded as comparatively efficient.

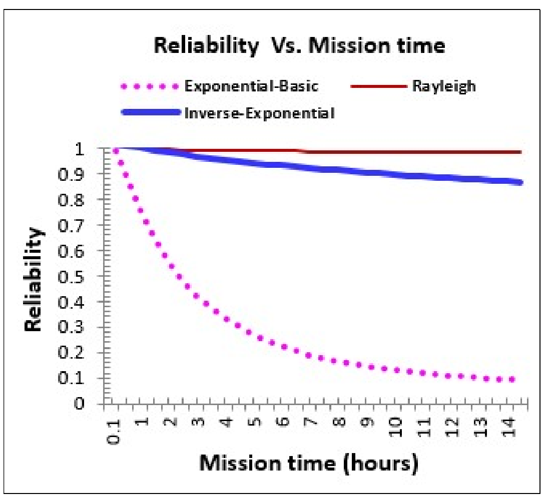

Figure 5: Performance Analysis by $\hat{R}(\tau)$.

Accordingly, the resulting reliability performance trends are depicted in Table 8. Also, as shown in Figure 5, $\hat{R}(\tau)$ takes values between 0 and 1, where larger values correspond to higher reliability of the model. For this reason, the Rayleigh model can be considered efficient in terms of reliability.

Table 8: $\hat{R}(\tau)$ Analysis.

Mission Time (hours) $\times 10^{-1}$	$\widehat{R}(\tau)$		
	Exponential -Basic	Inverse-Exponential	Rayleigh
0.1	1.03842	1.01879	1.02713
0.5	0.89855	1.01103	1.01742
1	0.75762	1.00185	1.00853
1.5	0.64567	0.99320	1.00227
2	0.55582	0.98503	0.99792
2.5	0.48300	0.97731	0.99490
3	0.42343	0.97000	0.99284
3.5	0.37429	0.9630	0.99144
4	0.33343	0.95649	0.99049
4.5	0.29919	0.95023	0.98986
5	0.27030	0.94427	0.98945
5.5	0.24576	0.93860	0.98918
6	0.22479	0.93318	0.98900
6.5	0.20676	0.92801	0.98889
7	0.19118	0.92306	0.98882
7.5	0.17764	0.91833	0.98877
8	0.16580	0.91380	0.98875
8.5	0.15547	0.90945	0.98873
9	0.14635	0.90528	0.98872
9.5	0.13829	0.90128	0.98871
10	0.13114	0.89743	0.98871
10.5	0.12478	0.89372	0.98871
11	0.11909	0.89016	0.98870
11.5	0.11400	0.88673	0.98870
12	0.10942	0.88342	0.98870
12.5	0.10530	0.88023	0.98870
13	0.10158	0.87715	0.98870
13.5	0.09820	0.87418	0.98870
14	0.09510	0.87130	0.98870
14.5	0.09238	0.86852	0.98870

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3.5. Reliability Performance Assessment

In this study, reference indices (MSE, R^2) were introduced to identify relatively efficient models in terms of goodness of fit, and the performance was evaluated based on the analysis of the property functions that assess reliability. Accordingly, the results obtained were utilized to compare and validate model performance, leading to the selection of the optimal model.

Table 9 shows the results of a comprehensive comparison and evaluation of the proposed models' performance, derived from the analyzed data. The results indicate that the Inverse-Exponential model demonstrates the highest level of performance [18].

Table 9: Results of Performance Assessment.

Model	Efficiency		Performance		
	MSE	R^2	m(t)	λ(t)	$\hat{R}(\tau)$
Exponential-Basic	Best	Best	Good	Good	Good
Inverse-Exponential	Best	Best	Best	Good	Good
Rayleigh	Worst	Worst	Worst	Worst	Best

Consequently, if the performance data of the models analyzed in Table 9 can be effectively applied during the initial development period, it will not only provide developers with essential design data but also serve as valuable reference data for preliminary testing required during the development preparation.

4. CONCLUSION

If developers make use of failure time data collected in the initial phase, they can design reliability prediction models that enable the estimation of failure occurrences in advance. Accordingly, failure time data accumulated during software operation can enhance product reliability and ultimately improve overall software quality. In this study, Weibull lifetime distributions, which are widely recognized as suitable for reliability, were implemented in NHPP-based reliability models. Consequently, the performance was comparatively evaluated, and their attributes were newly assessed.

The findings of this study are summarized as follows.
First, the examination of reference criteria (MSE, R^2) for determining model efficiency showed that the Inverse-Exponential model produced the highest level of fit.

Second, assessment of the performance properties (m(t), λ(t)) confirmed that the Inverse-Exponential model had the greatest efficiency, demonstrating superior true value prediction accuracy and the lowest failure intensity.

Third, the future reliability assessment revealed that, although the proposed models generally showed a time-dependent decrease in reliability, the Rayleigh model continually achieved the greatest reliability level, supporting its proven effectiveness.

The Inverse-Exponential model demonstrated superior performance compared to the other models evaluated. Accordingly, this study provides not only an algorithmic approach for analyzing performance data required by developers in the early stages of software development but also the fundamental data associated with such analyses. Therefore, future research should focus on identifying models optimized for specific areas of the software industry and conducting a more in-depth investigation into reliability-related properties.

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REFERENCES

[1] T. J. Yang, "Performance Analysis on the Reliability Attributes of NHPP Software Reliability Model Applying Exponential and Inverse-Exponential Lifetime Distribution", *Journal of Theoretical and Applied Information Technology,* Vol. 100, No. 22, 2022, pp. 6645-6656.

[2] T. J. Yang, "A Study on the Reliability Performance Analysis of Finite Failure NHPP Software Reliability Model Based on Weibull Life Distribution", *International Journal of Engineering Research and Technology,* Vol. 12, No. 11, 2019, pp. 1890-1896.

[3] K. Y. Song, I. H. Chang and H. Pham, "A Software Reliability Model with a Weibull Fault Detection Rate Function Subject to Operating Environments", *Applied Sciences,* Vol. 7, No. 10, 2017, pp. 983.

[4] H.C. Kim and J.B. Kim, "A Performance Comparative Evaluation for NHPP Software Reliability Model Considering Makeham and

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Extended Weibull Distribution", *Journal of Engineering and Applied Sciences,* Vol. 12, Special Issue 10, 2017, pp. 8909-8912.

[5] D. D. Hanagal and N. N. Bhalerao, "Modelling on Generalized Extended Inverse Weibull Software Reliability Growth Model", *Journal of Data Sciences,* Vol. 17, No. 3, 2019, pp. 575-592.

[6] J. Wang, H. Geng, and P. Lai, "A Software Reliability Model for Open Source Big Data Systems Based on Weibull-Weibull Distribution", *Scientific Reports 15,* Article number : 14670, 2025.

[7] T. J. Yang and J. G. Park, "A Comparative study of the Software NHPP Based on Weibull Extension Distribution and Flexible Weibull Extension Distribution", *International Journal of Soft Computing,* Vol. 11, No. 4, 2016, pp. 276-281.

[8] H. C. Kim, "A Comparative Study on Reliability Attributes of Software Reliability Model Dependent on Minimax and Lomax Lifetime Distribution", *Journal of Engineering and Applied Sciences,* Vol. 14, No. 24, 2019, pp. 9677-9681.

[9] A. L. Goel, and K. Okumoto "Time-dependent Fault Detection Rate Model for Software and Other Performance Measures", *IEEE Transactions on Software Engineering,* Vol. 28, 1978, pp. 206-211.

[10] T. J. Yang, "Comparative Analysis on the Reliability Performance of NHPP Software Reliability Model Applying Exponential-type Lifetime Distribution", *International Journal of Performance Engineering,* Vol. 18, No. 10, 2022, pp. 679-689.

[11] H. C. Kim, "The Comparative Study for Statistical Process Control for Software Reliability Property Utilizing Inverse Rayleigh and Rayleigh Distribution", *Journal of Engineering and Applied Sciences,* Vol. 13, No. 1, 2018, pp. 2500-2504.

[12] Y. Hayakawa, G. Telfar "Mixed Poisson-type processes with application in software reliability", *Mathematical and Computer Modelling,* Vol. 31, 2000, pp. 151-156.

[13] T. J. Yang, "Performance Evaluation of NHPP Software Reliability Model Applying Gamma Family Lifetime Distribution", *ARPN Journal of Engineering and Applied Science,* Vol. 18, No. 10, 2023, pp. 1171-1180.

[14] T. J. Yang, "Comparative Study on the Performance Evaluation of Infinite Failure NHPP Software Reliability Model with Log-Type Distribution Property", *ARPN Journal of Engineering and Applied Sciences,* Vol. 17, No. 11, 2022, pp. 1209-1218.

[15] T. H. Park, "A Comparative Analysis on the Performance Attributes of a Software Development Cost Model Based on Weibull Lifetime Distribution", *Journal of Theoretical and Applied Information Technology,* Vol. 101, No. 13, 2023, pp. 5229-5239.

[16] Y. C. Ra, "A Comparative Analysis on the Performance of Finite Failure NHPP Software Reliability Model Based on Rayleigh-type Lifetime Distribution", *Journal of Theoretical and Applied Information Technology,* Vol. 99 , No. 24, 2021, pp. 6162-6172.

[17] S. K. Park, "Comparative Analysis on the Reliability Attributes of Finite Failure NHPP Software Reliability Model with Exponential Distribution Characteristics", *Journal of Theoretical and Applied Information Technology,* Vol. 100, No. 18, 2022, pp. 5220-5231.

[18] H. J. Bae, "Performance Assessment of NHPP Software Reliability Models Applying PARETO-type Lifetime Distribution Properties", *Journal of Theoretical and Applied Information Technology,* Vol. 103, No. 7, 2025, pp. 2857-2867.