AN INTEGRATED RELIABILITY MODEL FOR SERIES-PARALLEL SYSTEMS: OPTIMIZING REDUNDANCY ALLOCATION WITH INTERNAL RATE OF RETURN USING HEURISTIC AND DYNAMIC PROGRAMMING APPROACHES

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ABSTRACT

The primary objective of reliability engineering is to guarantee that systems and components carry out their intended duties in a consistent manner over a predetermined amount of time and under predetermined conditions. Within the realm of reliability theory, this model is responsible for optimizing system dependability through the strategic allocation of redundancy, all the while balancing limitations like as cost, weight, and volume in series-parallel configurations. The purpose of this study is to investigate the impact that various constraints, specifically weight, volume, dimensions, and spatial limitations, have on the improvement of system reliability. More specifically, this research focuses on spare components for standard drilling machines, which include mechanical elements such as pulleys and gears that enable motion transmission and load management. Utilizing the Lagrangean multiplier method, a system that has an integrated redundant reliability series-parallel configuration is methodically designed and evaluated. This results in the production of real-valued solutions for critical parameters such as component quantities, component reliability, stage reliability, and overall system reliability. For the purpose of obtaining integer answers, the research utilized the heuristic algorithm method as well as dynamic programming approaches. As a result, the analytical precision and importance of the dependability analysis were significantly improved.

Keywords: *IRR Model, Series-Parallel Configuration, System Reliability, LAM Approach, HAM Approach, DMM Approach*

1. INTRODUCTION

In classical reliability theory, systems and their components are often limited to two distinct states: operational or failed. This binary perspective, however fundamental, constrains the analytical scope by neglecting intermediate situations. The framework of multi-state systems, however, enhances this study by permitting both the entire system and its individual components to exist along a continuum of states. This expanded range of options enables a more intricate and comprehensive knowledge of reliability, encompassing the diverse levels of performance and deterioration that arise in real-world situations. The Integrated Redundancy Allocation Problem (IRAP) has been extensively studied to improve system reliability by optimally allocating redundant components under resource constraints. This review summarizes the key contributions in this field, focusing on methodologies, problem formulations, and solution approaches.

Mishra, K. B. (1972), established a mathematical framework designed to enhance system reliability within linear limitations. The system consisted of multiple phases, each employing parallel redundancy. The model was transformed into a saddle point problem using Lagrange multipliers, and Newton's method was utilized to solve the resulting equations, incorporating adjustments to improve computer efficiency. The model was further developed into a multistage decision-making framework utilizing the Maximum Principle, providing an effective and easily implementable approach that guaranteed convergence while minimizing computational effort.

Ei-Neweihi, E. (1986), investigated the application of Schur-convex functions and majorization methods to enhance component distribution in parallel-series and series-parallel systems, with the objective of augmenting overall system reliability. In parallel-series systems, the optimal allocation was entirely dictated by the hierarchy of component reliabilities, whereas in series-parallel systems, a partial ordering of allocations was established to facilitate the optimization process. The research demonstrated that these issues might be restructured as integer linear programming models, yielding precise solutions in certain instances and delivering significant insights in others via the utilization of Schur function techniques. Coit, D.W. (2001) proposed an approach for determining the optimal design configurations in non-repairable series-parallel systems that utilize cold-standby redundancy. This methodology included variable component hazard rates and suboptimal switching mechanisms. Unlike earlier models that assumed flawless switching and exponential time-to-failure, Coit's method allowed for the option of different components within each subsystem, with time-to-failure represented by an Erlang distribution. The methodology more correctly modeled the engineering design issues, as effectively proven on a complex system comprising 14 subsystems. This integer programming-based approach yielded substantial enhancements in the computation of system reliability.

Elegbede et al. (2003), analyzed reliability allocation for parallel-series systems to minimize costs. They established that reliability of redundant components within a subsystem must be identical under typical cost functions. The ECAY algorithm was introduced for efficient reliability and redundancy allocation, outperforming Lagrange multiplier-based methods in both cost and computational efficiency. Konak (2004) proposed a novel formulation to maximize the minimum subsystem reliability in series-parallel systems. The method integrated integer programming for component mixing and utilized sequential max-min subproblems. This approach proved beneficial for balancing subsystem reliability, especially under tight resource constraints.

Kumar et al. (2009), emphasized multilevel redundancy, highlighting the inefficiencies of single-level approaches. Using a hierarchical genetic algorithm (HGA), they optimized modular redundancy in series and series-parallel configurations. The study revealed significant reliability improvements with modular designs compared to component-level redundancy. Ebrahim Nezhad et al. (2011), developed a multi-objective RAP model to simultaneously maximize system reliability and profit. This work introduced mixed active and standby strategies as decision variables, enabling more realistic and flexible designs. Murat (2012), advanced the multi-objective RAP field by employing a decomposition-based method. The approach ensured exact Pareto-optimal solutions, addressing limitations of meta-heuristic techniques such as incomplete Pareto fronts.

Twum and Aspinwall (2013), provided a comprehensive review of reliability optimization models, emphasizing gaps in redundancy allocation strategies. They noted a lack of multi-objective models and the dominance of redundancy-based approaches, calling for more exploration of reliability allocation strategies. Hamadani (2014) introduced the K-mixed strategy, which combines active and standby redundancies. This method was shown to enhance reliability significantly compared to traditional mixed strategies, particularly when coupled with genetic algorithms. Caserta and VoB (2015), reformulated RAP as a multiple-choice knapsack problem, solved using a branch-and-cut algorithm. Their method provided optimal solutions quickly for benchmark problems. Similarly, Soltani (2015), extended RAP to incorporate supplier selection, maximizing reliability, profit, and warranty length through compromise programming.

Guilani et al. (2017), addressed RAP for k-out-of-n systems with tri-state components, introducing organizational and technical activities to enhance reliability. Using SPEA-II and NSGA-II algorithms, they validated the efficiency of the bi-objective model. Aghaei et al. (2017) further improved k-out-of-n systems by treating redundancy strategies as decision variables. This novel approach combined integer programming with genetic algorithms for enhanced optimization.

Sridhar Akiri et al. (2021), conducted a comprehensive study on the design, analysis, and optimization of an integrated coherent redundant reliability system, a subject that had not been before recorded. The system's architecture was initially assessed using the Lagrangean multiplier, yielding integer solutions to improve dependability via integer and dynamic programming techniques, hence ensuring practical applicability. Sridhar Akiri et al. (2022), outlined the expanding realms of systems and software modeling, including intelligent synthetic entities, human-machine interfaces, and software systems. Their study offered practical tools for optimization, simulation, and reliability modeling, focusing on resource allocation and cost modeling for complex systems, thus assisting students, researchers, and industry professionals.

Srinivasa Rao Velampudi et al. (2023), conducted a study incorporating extraneous reliability into structured systems through the utilization of Lagrangian multipliers and dynamic programming methods. A heuristic approach was employed to provide an integer solution, improving system efficiency by assessing factors such as cost, size, and load. The proposed method aimed to enhance system performance by evaluating phase reliabilities and factor efficiencies, with results illustrated using a numerical example. Srinivasa Rao Velampudi et al. (2024), conducted a case study on the Muffle Box Furnace to improve system efficiency. The research utilized Lagrangean techniques to calculate the price, weight, and volume elements for various system configurations, leading to the development of a United Reliability Model (URM). The study integrated value constraints into IRR models, emphasizing the relationship between component cost and reliability, while also incorporating weight and volume as additional restrictions.

Bhavani Kapu (2024), introduced the Integrated Redundant Reliability Model (IRRM), enhancing system reliability through a parallel-series configuration. The model, designed for critical systems, utilized Lagrangian methods and modifications of the Newton-Raphson technique to improve component efficiency, phase reliability, and overall system performance, providing support for single-phase AC synchronous generators and ensuring operational continuity despite subsystem failures. Bhavani Kapu (2024), proposed the Integrated Redundant Reliability Model (IRRM), utilizing a parallel-series configuration to improve system reliability by increasing component efficiency and reducing system vulnerabilities. The model, designed for critical systems, employed Lagrangian methods and simulation techniques to improve system reliability and efficiency through integrated redundancy strategies.

In the present paper, the authors performed a comprehensive study on Series-Parallel configurations in relation to Integrated Redundant Reliability (IRR) Models, emphasizing redundant reliability arrangements. This research utilized a comprehensive case study focused on spare parts frequently utilized in typical drilling machine, including electric motors, drill jigs and pulleys or gears. The study provided substantial insights into design concerns and the development of integrated reliability systems, ultimately advancing both engineering practice and the field of reliability theory.

This study investigated a series-parallel configuration by developing an IRR and employing the conventional Lagrange multipliers method to get real-valued solutions, considering both rounded and unrounded outcomes. The heuristic algorithm approach and dynamic programming method was introduced as an innovative technique for generating integer values, enabling a comparison with the Lagrangean method and providing scientifically rigorous solutions. This methodology sought to preserve the necessary quantity of components (x_{com}) at each phase while improving overall system dependability ($R_{Sys\ Re}$).

2. APPROACHES:

Considerations and Symbols:

- Uniformity is assumed among elements within each stage, signifying that all elements share an equivalent level of reliability.
- Statistical independence is attributed to all elements, implying that the failure of one element exerts no influence on the functionality of other elements within the structure.

$R_{Sys\ Re}$: Systems Efficiency in Series-Parallel Configuration

$R_{Ph\ Re}$: Process Phase Efficiency 'pq', $0 < R_{Ph\ Re} < 1$

$r_{Com\ Re}$: Component Efficiency in the Phase 'q''; where $0 < r_{Com\ Re} < 1$

x_{com}: Numerous of items in Phase 'pq'

C_{com}: Item's-Price factor for each element in the phase 'pq'

W_{com}: Item's-Weight factor for each element in the phase pq'
V_{com}: Item's-Volume factor for each element in the phase 'pq'
$C_{Al\,Co}$: Maximum permissible Component's-Price
$W_{Al\,Wo}$: Maximum permissible Component's-Load
$V_{Al\,Vo}$: Maximum permissible Component's-Size
LMT Lagrange Multiplier Technique
HPA Heuristic Procedure Approach
DPT Dynamic Programming Technique
IRRM Integrated Reliability and Redundancy Model
a_q, d_q, e_q, g_q, i_q, k_q are Constants.

3. THE MODEL: AN ANALYSIS OF IT

The system's dependability concerning the given value function

$$\text{Maximize } R_{Sys\,Re} = 1 - \prod_{p=1}^{m}\left[1 - \prod_{q=1}^{n} R_{Ph\,Re}\right] \quad (1)$$

The subsequent correlation between value and efficiency is employed to determine the value coefficient of each unit in the phase 'pq'.

$$r_{Com\,Re} = \text{Sin}\left[\frac{C_{qc}}{a_q}\right]^{\frac{1}{d_q}} \quad (2)$$

Where C_{qc} a_p and $\frac{1}{d_q} > 0$ and C_{qc} is cost co-efficient a_q, d_q are constants.
Therefore

$$C_{pc} = a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} \quad (3)$$

Similarly,

$$W_{wc} = e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} \quad (4)$$

$$V_{vc} = i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} \quad (5)$$

Since component's-price is linear in 'pq',

$$\sum_{q=1}^{n} C_{pc}\, x_{com} \le C_{Al\,Co} \quad (6)$$

Similarly, component's-weight and component's-volume are also linear in 'pq'.

$$\sum_{q=1}^{n} W_{wc}\, x_{com} \le W_{Al\,Wo} \quad (7)$$

$$\sum_{q=1}^{n} V_{vc}\, x_{com} \le V_{Al\,Vo} \quad (8)$$

From (3), (4), (5) we get

$$\sum_{q=1}^{n} a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} x_{com} - C_{Al\,Co} \le 0 \quad (9)$$

$$\sum_{q=1}^{n} e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} x_{com} - W_{Al\,Wo} \le 0 \quad (10)$$

$$\sum_{q=1}^{n} i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} x_{com} - V_{Al\,Vo} \le 0 \quad (11)$$

The transformed equation through the relation

$$x_{com} = \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} \quad (12)$$

Where $R_{Sys\,Re} = \prod_{p=1}^{m}\left[1 - (1 - r_{Com\,Re})^{x_{com}}\right]$
Subject to the constraints

$$\sum_{q=1}^{n} a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - C_{Al\,Co} \le 0 \quad (13)$$

$$\sum_{q=1}^{n} e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - W_{Al\,Wo} \le 0 \quad (14)$$

$$\sum_{q=1}^{n} i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - V_{Al\,Vo} \le 0 \quad (15)$$

Non-negative restrictions 'pq' ≥ 0
A Lagrangean function is defined as

$$Z = R_{Sys\,Re} + \mu_1\left[\sum_{q=1}^{n} a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - C_{Al\,Co}\right] + \mu_2\left[\sum_{q=1}^{n} e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - W_{Al\,Wo}\right] + \mu_3\left[\sum_{q=1}^{n} i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}} - V_{Al\,Vo}\right]$$

Utilizing the Lagrangean function enables the identification of the optimal point and its separation by where $R_{Ph\,Re}$, $r_{Com\,Re}$, μ_1, μ_2, μ_3 are idle points.

$$\frac{\partial Z}{\partial R_{Sys\,Re}} = 1 + \mu_1\left[\sum_{q=1}^{n} a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} \frac{1}{R_{Ph\,Re}\, Log\, r_{Com\,Re}}\right] + \mu_2\left[\sum_{q=1}^{n} e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} \frac{1}{R_{Ph\,Re}\, Log\, r_{Com\,Re}}\right] + \mu_3\left[\sum_{q=1}^{n} i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} \frac{1}{R_{Ph\,Re}\, Log\, r_{Com\,Re}}\right] \quad (17)$$

$$\frac{\partial Z}{\partial r_{Com\,Re}} = \mu_1\left[\sum_{q=1}^{n} a_q \,\text{Sin}^{-1}(r_{Com\,Re})^{d_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}}\left(\frac{d_q}{\text{Sin}^{-1} r_{Com\,Re}\left(\sqrt{(1-(r_{Com\,Re})^2)}\right)} - \frac{1}{R_{Ph\,Re}(Log\, r_{Com\,Re})}\right)\right] + \mu_2\left[\sum_{q=1}^{n} e_q \,\text{Sin}^{-1}(r_{Com\,Re})^{g_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}}\left(\frac{g_q}{\text{Sin}^{-1} r_{Com\,Re}\left(\sqrt{(1-(r_{Com\,Re})^2)}\right)} - \frac{1}{R_{Ph\,Re}(Log\, r_{Com\,Re})}\right)\right] + \mu_3\left[\sum_{q=1}^{n} i_q \,\text{Sin}^{-1}(r_{Com\,Re})^{k_q} \frac{Log\, R_{Ph\,Re}}{Log\, r_{Com\,Re}}\left(\frac{k_q}{\text{Sin}^{-1} r_{Com\,Re}\left(\sqrt{(1-(r_{Com\,Re})^2)}\right)} - \frac{1}{R_{Ph\,Re}(Log\, r_{Com\,Re})}\right)\right] \quad (18)$$

$\frac{\partial Z}{\partial \mu_1} = \sum_{q=1}^{n} a_q \ \mathrm{Sin}^{-1} \left(r_{\mathrm{Com\,Re}}\right)^{d_q} \frac{Log\ R_{\mathrm{Ph\,Re}}}{Log\ r_{\mathrm{Com\,Re}}} - C_{Al\,Co}$ (19)

$\frac{\partial Z}{\partial \mu_2} = \sum_{q=1}^{n} e_q \ \mathrm{Sin}^{-1} \left(r_{\mathrm{Com\,Re}}\right)^{g_q} \frac{Log\ R_{\mathrm{Ph\,Re}}}{Log\ r_{\mathrm{Com\,Re}}} - W_{Al\,Wo}$ (20)

$\frac{\partial Z}{\partial \mu_3} = \sum_{q=1}^{n} i_q \ \mathrm{Sin}^{-1} \left(r_{\mathrm{Com\,Re}}\right)^{k_q} \frac{Log\ R_{\mathrm{Ph\,Re}}}{Log\ r_{\mathrm{Com\,Re}}} - V_{Al\,Vo}$ (21)

Where μ_1, μ_2 and μ_3 are Lagrangean multipliers.

Utilizing the Lagrangean approach, we ascertain the quantity of elements in each phase (x_{com}), identify the optimal reliability of components ($r_{\mathrm{Com\,Re}}$), calculate the reliability of each stage ($R_{\mathrm{Ph\,Re}}$), and assess the overall structural reliability ($R_{\mathrm{Sys\,Re}}$). This method provides a definitive numerical solution concerning the component's cost, weight, and volume.

4. PARAMETERS OF THE CASE STUDY:

This study optimizes mechanical system parameters assuming cost, weight, and volume factors directly affect system efficiency. Electronic systems may not make this assumption. For mechanical systems, it is crucial to evaluate component level reliability ($r_{\mathrm{Com\,Re}}$), stage reliability ($R_{\mathrm{Ph\,Re}}$), element quantity per stage (x_{com}) and structural accuracy ($R_{\mathrm{Sys\,Re}}$). The optimum component allocation in series-parallel systems is reviewed extensively, revealing precise algorithms, redundancy distributions, and replaceable component techniques. We learn more about reliability engineering in complex systems from this research. A specialized machine for assembling ordinary drilling machines is examined in this study for structural accuracy. Figure 1 shows a common drilling machine schematic.

In real-world applications, various essential components play a significant role in ensuring system performance and reliability. These include, but are not limited to, electric motors, which are crucial for mechanical operations; drill jigs, which are indispensable in manufacturing processes for maintaining precision and accuracy; and mechanical elements like pulleys or gears, which facilitate motion transmission and load management. These components are often evaluated and optimized using an integrated redundant reliability model to enhance their operational efficiency, minimize failures, and ensure sustained functionality over time. The cost of drilling machine which is using in our case problem typically ranges from \$700 to \$1,200 USD, influenced on their capacity and features. Weights often vary from 110 to 220 lbs, although the volume of a normal oil burner may range from 30 to 80 m^2, contingent upon the brand and kind.

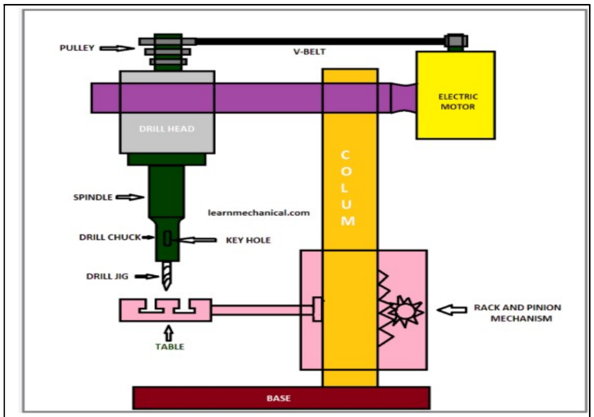

Figure 1: Typical Drilling Machine

4.1 Parameters of the Case Problem:

The constants necessary for the case problem are presented in Table 1.

Table 1: *Preset Constant Values for Parameters of Value, Load, and Size in Series-Parallel Configuration Systems*

Phase	Constituents of Value		Constituents of Load		Constituents of Dimension	
	a_q	d_q	e_q	g_q	i_q	k_q
I	700	0.92	110	0.91	30	0.95
II	1000	0.94	160	0.92	55	0.96
III	1200	0.96	220	0.94	80	0.98

The tables 2, 3 and 4 below display the structural efficiency, as well as the efficiency of each factor, phase, and number of factors in each stage.

4.2 Comprehensive Analysis of Component and Stage Reliabilities for Price, Load and Size Constraints Utilizing the Lagrange Multiplier Method Without Rounding Techniques in Series-Parallel Configuration Systems:

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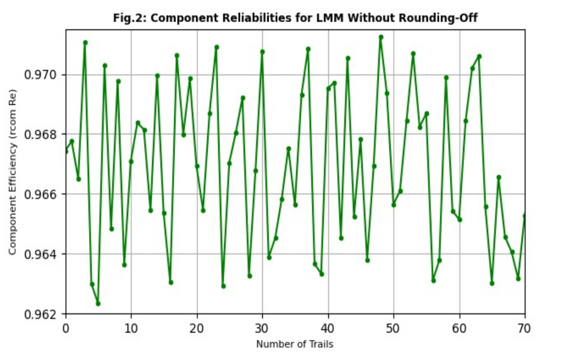
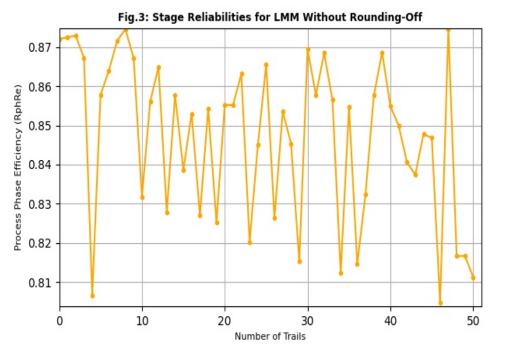
Figures 2 and 3 illustrate the component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) of price, load and volume constraints obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, load and size.

The Component ($r_{Com\ Re}$) and Process Phase Efficiencies ($R_{Ph\ Re}$) by Using LMT Without Rounding-Off





4.3 Comprehensive Analysis of Component Price Constraints Utilizing the Lagrange Multiplier Method Without Rounding Techniques in Series-Parallel Configuration Systems

Here in Table 2 delineates the value-related efficiency design, the authors selected the optimal component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) based on price constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total cost and overall system efficiency.

Table 2: Analysis of Price Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	a_a	d_a	$r_{Com\ Re}$	Log $r_{Com\ Re}$ $r_{\beta j}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	C_{pc}	$C_{pc}\ x_{com}$
I	700	0.92	0.9621	-0.0168	0.8039	-0.0948	5.65	851	4810
II	1000	0.94	0.9674	-0.0144	0.8745	-0.0582	4.05	1336	5406
III	1200	0.96	0.9714	-0.0126	0.8215	-0.0854	6.78	1600	10845
Ultimate Value									21061
Efficiency of System ($R_{Sys\ Re}$)									0.9752

4.3 Comprehensive Analysis of Component Load Constraints Utilizing the Lagrange Multiplier Method Without Rounding Techniques in Series-Parallel Configuration Systems

Here in Table 3 delineates the load-related efficiency design, the authors selected the optimal component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) based on price constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total load and overall system efficiency.

Table 3: Analysis of Load Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	e_a	g_a	$r_{Com\ Re}$	Log $r_{Com\ Re}$ $r_{\beta j}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	W_{wc}	$W_{wc}\ .x_{com}$
I	110	0.91	0.9621	-0.0168	0.8039	-0.0948	5.65	134	755.74
II	160	0.92	0.9674	-0.0144	0.8745	-0.0582	4.05	214	864.69

III	220	0.94	0.9714	-0.0126	0.8215	-0.0854	6.78	293	1987.51
Ultimate Load									3607.94
Efficiency of System ($R_{Sys\ Re}$)									0.9752

4.4 Comprehensive Analysis of Component Size Constraints Utilizing the Lagrange Multiplier Method Without Rounding Techniques in Series-Parallel Configuration Systems

Here in Table 4 delineates the size-related efficiency design, the authors selected the optimal component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) based on size constraints from these iterations. Using these optimal values, the number of components required for each phase was determined, along with the corresponding total load and overall system efficiency.

Table 4: Analysis of Size Constraints in Series-Parallel Configuration Systems Using the LMT

Phase	i_q	k_q	$r_{Com\ Re}$	Log $r_{Com\ Re}$ $r_{\beta j}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	V_{Vc}	$V_{vc} \cdot x_{com}$
I	30	0.95	0.9621	-0.0168	0.8039	-0.0948	5.65	37	206.32
II	55	0.96	0.9674	-0.0144	0.8745	-0.0582	4.05	74	297.46
III	80	0.98	0.9714	-0.0126	0.8215	-0.0854	6.78	107	723.21
Ultimate Dimension									1226.99
Efficiency of System ($R_{Sys\ Re}$)									0.9752

5. OPTIMIZATION OF EFFICIENCY THROUGH THE APPLICATION OF THE LAGRANGE MULTIPLIER METHOD

The efficiency of system consolidates the $'pq'$ values as integers by rounding $'pq'$ to the nearest whole number, while the permissible outcomes for worth, load, and size are enumerated in the tables. Compute the variation attributable to value, load, dimensions, and construction capacity (both prior to and subsequent to including rounding-off $'pq'$ to the nearest integer) to acquire data.

5.1 Comprehensive Analysis of Components Like Price, Load and Size Constraints Utilizing the Lagrange Multiplier Technique With-Rounding-Off in Series-Parallel Configuration Systems

Figures 4 and 5 illustrate the component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) of price, load and size constraints obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, weight, and size.

The Component ($r_{Com\ Re}$) and The Process Phase Efficiencies ($R_{Ph\ Re}$) by Using LMT With Rounding-Off

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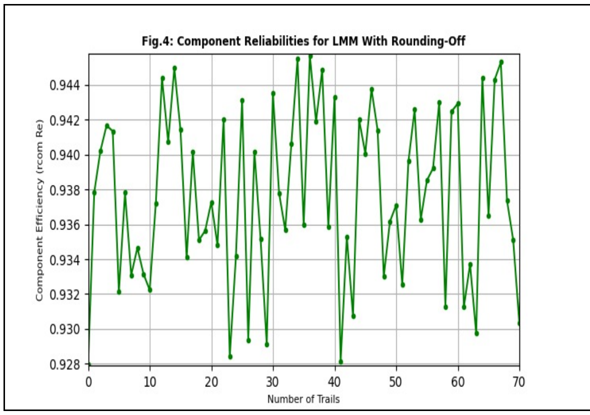

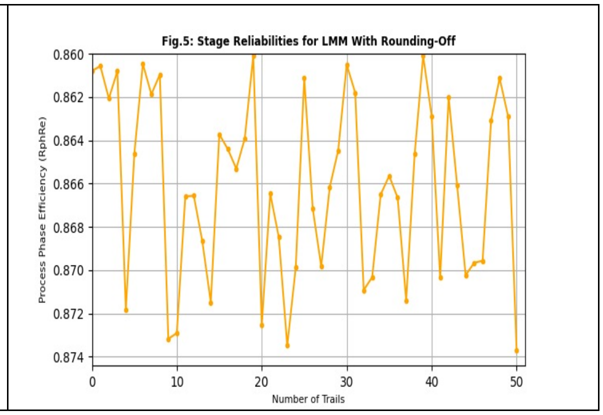

5.2 Efficiency Design Analysis Utilizing the Lagrange Multiplier Method in Relation to Value, Load, and Size Parameters Including Rounding-off Techniques in Series-Parallel Configuration Systems

Table 5: *Analysis of Efficiency Design Concerning Value, Load, and Size Constraints Utilizing the Lagrange Multiplier Method Including Rounding-off Techniques, Presented in the Following Table*

Phase	$r_{Com\ Re}$	$R_{Ph\ Re}$	x_{com}	C_{pc}	$C_{pc} \cdot x_{com}$	W_{wc}	$W_{wc} \cdot x_{com}$	V_{vc}	$V_{vc} \cdot x_{com}$
I	0.9281	0.8625	6	871	5225	137	821	37	224
II	0.9345	0.8745	4	1362	5446	218	871	75	300
III	0.9457	0.8602	7	1624	11370	298	2083	108	758
Total Worth, Load and Size				22041		3775		1283	
Efficiency of System (R_{StRe})								0.9799	

5.2.1 Variation in the Price-Component by Using LMM = 04.65%
5.2.2 Variation in the Load-Component by Using LMM = 04.63%
5.2.3 Variation in the Volume-Component by Using LMM = 04.56%
5.2.4 Variation in Change of Efficiency in System by Using LMM = 00.48%

6. HEURISTIC ALGORITHM APPROACH:

Heuristic problem solving is identifying a set of rules or procedures that yield satisfactory solutions to a particular situation. A heuristic approach is typically employed instead of an optimization strategy when the problem does not fit into any classifications and is therefore not suitable for established solution methods. Even when the issue does not fit into a recognized category, there may be resource constraints, such as computational time and data needs, that render the optimization strategy unsuitable. The subsequent list enumerates problem characteristics that may suggest the use of heuristics. Problems lacking proven algorithmic solutions are termed ill-structured.

A well-defined problem possesses the following attributes:

1. All pertinent information regarding the issue can be encapsulated in a suitable model.
2. The model must encompass all viable solutions.
3. An algorithm exists for determining the optimal solution to the model.
4. All necessary data should be economically feasible to collect.

6.1 Heuristic Algorithm

In computer science, a Heuristic Algorithm, or Heuristic, is an algorithm that generates an acceptable solution to a problem in several actual situations, akin to a general Heuristic, although lacks formal evidence of its correctness. Alternatively, it may be accurate, although it may not be demonstrated to yield an optimal solution or to utilize resources judiciously. Heuristics are often employed when there is no known method to ascertain an optimal solution within the given restrictions or in general.

Two primary objectives in computer science are to identify algorithms with demonstrably efficient run times and those that yield provably optimal solution quality. A heuristic is an algorithm that forsakes one or both of these objectives. It typically identifies satisfactory answers; but, there is no evidence to suggest that these solutions cannot become exceedingly poor. Additionally, it generally operates at a reasonable speed, although there is no assurance that this will consistently hold true.

The author of this suggested work developed novel heuristic processes for the mathematical function under examination to enhance system reliability in series-parallel configuration redundant reliability systems with numerous restrictions. The technique for the Heuristic algorithm is outlined below.

6.2 Proposed Heuristic Algorithm

Step 1: Define and input the required initial parameters.
Step 2: Specify the maximum allowable number of components, denoted as n.
Step 3: Begin by setting the number of components in the first stage to one, and compute the system's cost (C_{com}), weight (W_{com}), and volume (V_{com}) for this stage.
Step 4: Assign the number of components in the second stage as one, and calculate the corresponding values of cost (C_{com}), weight (W_{com}), and volume (V_{com}) for this stage.
Step 5:

a. Set the number of components in the third stage to one, and determine the system's cost (C_{com}), weight (W_{com}), and volume (V_{com}) for this stage.
b. Add the values of cost (C_{com}), weight (W_{com}), and volume (V_{com}) from all three stages.
c. Compute the system reliability ($R_{Sys\ Re}$).
d. Verify if the constraints are satisfied.

Step 6:

a. If the constraints are met, display the number of components and the system reliability (R_{StRe}) for the corresponding configuration.
b. If the constraints are not satisfied, increase the number of components in the third stage by one and return to **Step 5**.
c. Repeat this process until the total number of components across all three stages reaches the maximum allowable value (n).

6.3 Heuristic Solution Approach

The heuristic approach for the proposed mathematical function is applied to establish and optimize integrated reliability models for redundant systems in a series-parallel configuration under multiple constraints. To achieve an optimized design, the case problem discussed in the preceding section is considered. The values of component reliabilities and the number of components in each stage are taken as inputs for applying the heuristic approach.

This approach is particularly effective in optimizing the design where the values of items are integers, making it highly practical for real-world applications. The various values related to cost, weight, and volume for the proposed integrated redundant reliability models, determined through the heuristic method, are analyzed and presented for the mathematical function under investigation in the following section.

6.4 The Component ($r_{Com\ Re}$) and Process Phase Efficiencies ($R_{Ph\ Re}$) by Using Heurist Algorithm Approach

Figures 6 and 7 illustrate the component efficiencies ($r_{Com\ Re}$) and process phase efficiencies ($R_{Ph\ Re}$) of size constraint obtained through approximately 50 iterations of a trial-and-error method using a MATLAB program. The program was developed to construct an integrated redundant reliability model based on a series-parallel configuration, taking into account constraints such as cost, weight, and size. .

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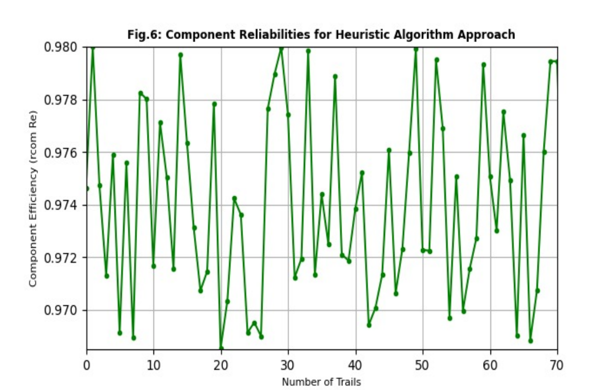
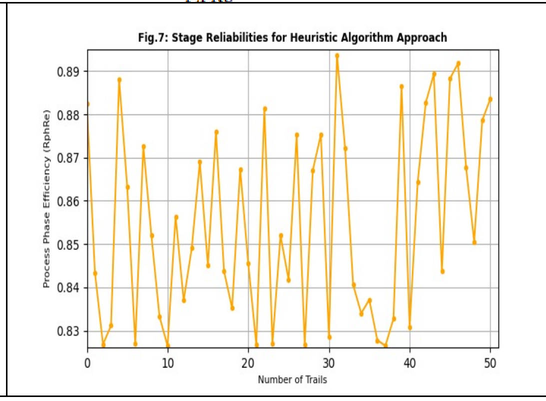
The Component ($r_{Com\ Re}$) and Process Phase Efficiencies ($R_{Ph\ Re}$) by Using HAA





6.5 In-Depth Analysis of Component Worth Constraints Utilizing the Heuristic Algorithm approach in Series-Parallel Configuration Systems

Table 6: *Detailed Analysis of Component Price, Weight and Size Constraints Utilizing a Heuristic Algorithm Approach in Series-Parallel Configuration Systems*

Phase	$r_{Com\ Re}$	$R_{Ph\ Re}$	x_{com}	C_{pc}	$C_{pc} \cdot x_{com}$	W_{wc}	$W_{wc} \cdot x_{com}$	V_{vc}	$V_{vc} \cdot x_{com}$
I	0.9687	0.8262	6	848	5091	133	800	36	218
II	0.9726	0.8948	4	1332	5331	213	853	73	293
III	0.9799	0.8675	7	1593	11150	292	2044	106	744
Total Worth, Load and Size				21570		3696		1225	
Efficiency of System (R_{StRe})								0.9824	

6.5.1 Variation in the Price-Component by Using HAA = 02.18%
6.5.2 Variation in the Load-Component by Using HAA = 02.13%
6.5.3 Variation in the Volume-Component by Using HAA = 04.73%
6.5.4 Variation in Change of Efficiency in System by Using HAA = 00.26%

Heuristics are used to create integrated reliability models for redundant systems with numerous constraints based on the proposed mathematical function. The Lagrangean method generates the present case problem inputs in the heuristic approach. The results show that the heuristic technique is most useful for real-life applications due to its ability to produce integer numbers for the number of components in each step (x_{com}). Traditional methods like integer or dynamic programming can yield integer results, but their calculation time grows exponentially with issue size (variables). This constraint is overcome by using the heuristic method to efficiently find integer solutions.

The primary disadvantage of the heuristic procedure is that, while it provides a solution, it is considered to be an approximate solution. Although it is a workable and practical solution, it may not necessarily be a closed or bounded one. This procedure is particularly useful in industrial applications for deriving near-optimal workable solutions, especially for problems with a large number of variables. At this stage, the author considered deriving an exact integer-valued solution for the proposed mathematical models, which led to the application of the dynamic programming technique. The results of this approach are presented in the subsequent section of the manuscript.

7. DYNAMIC PROGRAMMING TECHNIQUE:

The Lagrangian method has certain limitations, such as the requirement to define the number of components ('pq') at each stage using exact values, which can be difficult to implement in practice. The common practice of truncating result values leads to changes in parameters such as worth, load, and size, thereby affecting system reliability and significantly influencing the efficiency of the model's design. To address this issue, the author suggests an alternative empirical approach, leveraging the dynamic programming technique to obtain integer solutions by utilizing the outputs of the Lagrangian method as input parameters for the dynamic programming process.

7.1 Optimizing System Reliability and Resource Constraints Using Dynamic Programming in Series-Parallel Configuration

In the context of series-parallel configurations using an integrated redundant model, dynamic programming can be employed to address optimization problems related to cost, load, size, and system reliability. By leveraging Python programming, the approach involves decomposing the primary problem into smaller sub-problems and systematically storing their solutions to eliminate redundant computations. This method is particularly effective when dealing with problems that exhibit overlapping substructures and have optimal sub-problem solutions. The following outlines a structured approach for dynamic programming:

1. **Problem Definition**: Clearly specify the problem to be addressed, including the objective to be optimized. Identify the relevant parameters, variables, and constraints, such as cost, load, and size in the integrated redundant model.
2. **Identify Optimal Substructure**: Decompose the problem into smaller sub-problems that mirror the structure of the original. These sub-problems should contribute to solving the overall system, including reliability and performance within the constraints of price and load.
3. **Formulate Recurrence Relations**: Establish recurrence relations that express the solution of each sub-problem in terms of the solutions to its smaller sub-problems. This creates a relationship between sub-problems, allowing for systematic solution building.
4. **Construct a Memorization Table**: To avoid repeated calculations, create a table or array that stores the solutions to subproblems as they are solved. This technique, known as memorization, starts with base cases that represent the simplest sub-problem solutions.
5. **Populate the Table**: Use the recurrence relations to fill in the table, starting from the smallest sub-problems and working upward. By reusing solutions from previously solved sub-problems, you efficiently build the solution to the larger problem.
6. **Retrieve the Final Solution**: Once the table is fully populated, the final solution to the original problem can be found by referencing the relevant entry in the table, providing insight into overall system reliability and optimization.
7. **Space Optimization (Optional)**: In some cases, memory usage can be optimized by only storing essential values from the table, particularly if only a subset of the solutions is required for further analysis.
8. **Bottom-Up Approach (Optional)**: Dynamic programming can be implemented either in a bottom-up or top-down manner. A bottom-up approach solves the smallest sub-problems first, iteratively building up to the complete solution, while a top-down approach employs recursion combined with memorization.
9. **Analyze Complexity**: Evaluate the time and space complexity of the dynamic programming solution. This analysis often reveals significant reductions in computational effort compared to more straightforward brute-force methods.
10. **Test and Validate**: Thoroughly test the dynamic programming solution using various inputs to ensure correctness. Verify that the results stored in the table align with the expected outcomes.

7.2 The Process Phase Efficiencies ($R_{Ph\,Re}$) by Using Dynamic Programming Technique

Dynamic programming is an effective technique applicable to a wide variety of optimization problems, including those related to reliability models, resource allocation, and system performance. Practicing this approach is essential for recognizing problems that can benefit from dynamic programming and for developing efficient, reliable solutions in Table 7, 8 and 9 in the below.

Table 7: Initial Stage of the Dynamic Programming Process

Phase–I($'pq'$)	Phase - I - Reliability ($\mathbf{R}_{Ph\ Re}$)
01	0.7625
02	0.7824
03	0.8111
04	0.8267
05	**0.8443**

Table 8: Dynamic Programming, Second Stage

Phase – II ($'pq'$)	Phase - II - Reliability ($\mathbf{R}_{Ph\ Re}$)							
04	0.7458	0.7564	0.7761	0.9045				
05	0.7944	0.7989	0.8054	0.9155	0.9035			
06	0.8243	0.8654	0.8425	0.8953	0.8881	0.9147		
07	0.8354	0.8954	0.9014	0.9115	0.9254	0.9354	0.9235	0.9452
08	0.8495	0.8547	0.8614	0.8832	0.8954	0.9014	0.9135	0.9254
09	0.8464	0.8847	0.8635	**0.9509**	0.9234	0.9167	0.9347	0.9365

Table 9: Dynamic Programming, Final Stage

Phase – III ($'pq'$)	Phase - III - Reliability ($\mathbf{R}_{Ph\ Re}$)							
05	0.8647	0.8874	0.8965					-
06	0.8647	0.8657	0.8947	0.9043				
07	0.8985	0.8999	0.9014	0.9135	0.9148			
08	0.9284	0.9145	0.9058	0.9167	0.9143	0.9268		
09	0.8983	0.9068	0.9236	0.9289	0.9248	0.9166	0.9052	
10	0.8147	0.9356	**0.9308**	0.9125	0.9142	0.9094	0.8952	0.9088

7.3 Results of DPT

The Lagrange multiplier method provided a continuous solution for the proposed Integrated

Redundant Reliability (IRR) Systems, which were modeled using series-parallel configurations. These configurations, commonly used in reliability engineering, combine components in both series and parallel arrangements to enhance system reliability. In the models under investigation, the method not only offered a real-valued (continuous) solution but also worked alongside the necessary integer solution required for practical application.

To further clarify and interpret the most critical findings of these models, the Dynamic Programming Approach was applied. This approach helped optimize decisions at each stage of the process, ensuring that both the series and parallel components of the IRR system were effectively managed to maximize reliability. The results of the mathematical function, which evaluated the performance and reliability of these systems, are summarized and presented in detail in Tables 10, 11 and 12. These tables illustrate how the combined use of Lagrange multipliers and dynamic programming contributed to solving the complex reliability problem posed by the series-parallel configuration in the IRR model.

7.4 In-Depth Analysis of Component Worth Constraints Utilizing the Dynamic Programming Technique in Series-Parallel Configuration Systems

Detailed information regarding the worth-related efficiency design can be found in Table 10.

Table 10: Detailed Analysis of Component Price Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems

Phase	a_a	d_a	$r_{Com\ Re}$	$Log\ r_{Com}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	C_{pc}	$C_{pc} \cdot x_{com}$
I	700	0.92	0.9722	-0.0122	0.8443	-0.0735	6	846	5079
II	1000	0.94	0.9875	-0.0055	0.9509	-0.0219	4	1322	5290
III	1200	0.96	0.9898	-0.0045	0.9308	-0.0311	7	1584	11090
Ultimate Worth									21459

7.5 In-Depth Analysis of Component Load Constraints Utilizing the Dynamic Programming Technique in Series-Parallel Configuration Systems

Detailed information regarding the load-related efficiency design can be found in Table 11.

Table11: Detailed Analysis of Component Load Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems

Phase	e_q	g_q	$r_{Com\ Re}$	$Log\ r_{Com\ Re}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	W_{wc}	$W_{wc} \cdot x_{com}$
I	110	0.91	0.9722	-0.0122	0.8443	-0.0735	6	133	798
II	160	0.92	0.9875	-0.0055	0.9509	-0.0219	4	211	846
III	220	0.94	0.9898	-0.0045	0.9308	-0.0311	7	290	2033
Ultimate Load									3677

7.6 In-Depth Analysis of Component Size Constraints Utilizing the Dynamic Programming Technique in Series-Parallel Configuration Systems

Detailed information regarding the size-related efficiency design can be found in Table 12.

Table 12: *Detailed Analysis of Component Size Constraints Utilizing a Dynamic Programming Approach in Series-Parallel Configuration Systems*

Phase	i_q	k_q	$r_{Com\ Re}$	$Log\ r_{Com\ Re}$	$R_{Ph\ Re}$	$Log\ R_{Ph\ Re}$	x_{com}	V_{vc}	$V_{vc}\ x_{com}$
I	30	0.95	0.9722	-0.0122	0.8443	-0.0735	6	36	218
II	55	0.96	0.9875	-0.0055	0.9509	-0.0219	4	73	291
III	80	0.98	0.9898	-0.0045	0.9308	-0.0311	7	106	739
Ultimate Dimension									1248
Efficiency of System ($R_{Sys\ Re}$)									0.9901

7.6.1 Variation in the Price-Component by Using DPT = 02.71%
7.6.2 Variation in the Load-Component by Using DPT = 02.67%
7.6.3 Variation in the Volume-Component by Using DPT = 02.81%
7.6.4 Variation in Change of Efficiency in System by Using DPT = 01.04%

8. COMPARATIVE STUDY:

In this study, the author compares the resultant values obtained from the Lagrangean multiplier method with those derived from the rounding-off method, the heuristic algorithm approach, and the dynamic programming method. The study focuses on the number of components, component reliabilities, stage reliabilities, and their respective overall system reliabilities.

8.1 A Comparative Study of Optimization Methods for Integrated Redundant Reliability in Series-Parallel Configuration Systems: Lagrange Multiplier with Rounding, Heuristic Algorithm Approach and Dynamic Programming for Value Improvement

Table13: *Results Correlating the Lagrange Multiplier Method Including a Rounding-off Technique, Heuristic Algorithm Approach and Dynamic Programming Approaches for Pricing in Series-Parallel Configuration Systems*

Ultimate Value		Including a Rounding Off			Heuristic Algorithm			Dynamic Programming		
Phase	x_{com}	$r_{Com\ Re}$	$R_{Ph\ Re}$	$C_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$C_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$C_{pc} \cdot x_{com}$
I	6	0.9589	0.8625	4026	0.9687	0.8262	5091	0.9722	0.8443	5079
II	4	0.9514	0.8745	4851	0.9726	0.8948	5331	0.9875	0.9509	5290

III	7	0.9523	0.8602	8084	0.9799	0.8675	11150	0.9898	0.9308	11090
Efficiency of System ($R_{Sys\ Re}$)		22041			21570			21459		
		LMT SE = 0.9799			HAA SE = 0.9824			DPT SE = 0.9901		

8.2 A Comparative Study of Optimization Methods for Integrated Redundant Reliability in Series-Parallel Configuration Systems: Lagrange Multiplier with Rounding, Heuristic Algorithm Approach and Dynamic Programming for Load Improvement

Table14: *Results Correlating the Lagrange Multiplier Method Including a Rounding-off Technique, Heuristic Algorithm Approach and Dynamic Programming Approaches for Loading in Series-Parallel Configuration Systems*

Ultimate Load		Including a Rounding Off			Heuristic Algorithm			Dynamic Programming		
Phase	x_{com}	$r_{Com\ Re}$	$R_{Ph\ Re}$	$W_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$W_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$W_{pc} \cdot x_{com}$
I	6	0.9589	0.8625	821	0.9687	0.8262	133	0.9722	0.8443	798
II	4	0.9514	0.8745	871	0.9726	0.8948	213	0.9875	0.9509	846
III	7	0.9523	0.8602	2083	0.9799	0.8675	292	0.9898	0.9308	2033
Efficiency of System ($R_{Sys\ Re}$)		3775			3696			3677		
		LMT SE = 0.9799			HAA SE = 0.9824			DPT SE = 0.9901		

8.3 A Comparative Study of Optimization Methods for Integrated Redundant Reliability in Series-Parallel Configuration Systems: Lagrange Multiplier with Rounding, Heuristic Algorithm Approach and Dynamic Programming for Size Improvement

Table15: *Results Correlating the Lagrange Multiplier Method Including a Rounding-off Technique, Heuristic Algorithm Approach and Dynamic Programming Approaches for Sizing in Series-Parallel Configuration Systems*

Ultimate Size		Including a Rounding Off			Heuristic Algorithm			Dynamic Programming		
Phase	x_{com}	$r_{Com\ Re}$	$R_{Ph\ Re}$	$V_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$V_{pc} \cdot x_{com}$	$r_{Com\ Re}$	$R_{Ph\ Re}$	$V_{pc} \cdot x_{com}$
I	6	0.9589	0.8625	224	0.9687	0.8262	218	0.9722	0.8443	218
II	4	0.9514	0.8745	300	0.9726	0.8948	293	0.9875	0.9509	291
III	7	0.9523	0.8602	758	0.9799	0.8675	744	0.9898	0.9308	739
Efficiency of System ($R_{Sys\ Re}$)		1283			1225			1248		
		LMT SE = 0.9799			HAA SE = 0.9824			DPT SE = 0.9901		

9. CONCLUSION:

In real-world applications, various essential components play a significant role in ensuring system performance and reliability. These include, but are not limited to, electric motors, which are crucial for mechanical operations; drill jigs, which are indispensable in manufacturing processes for maintaining precision and accuracy; and mechanical elements like pulleys or gears, which facilitate motion transmission and load management. These components are often evaluated and optimized using an integrated redundant reliability model to enhance their operational efficiency, minimize failures, and ensure sustained functionality over time. The cost of drilling machine which is using in our case problem typically ranges from $700 to $1,200 USD, influenced on their capacity and features. Weights often vary from 110 to 220 lbs, although the volume of a normal oil burner may range from 30 to 80 m^2, contingent upon the brand and kind.

In the present manuscript, the author presents a comprehensive integrated redundant reliability model that employs a series-parallel configuration to effectively determine component reliability, stage reliability, and the requisite number of components, thereby enhancing overall system reliability. The study focuses on a typical drilling machine, which consists of a multitude of interdependent components. Among these, the author critically examines key elements such as the electric motors, drill jigs, and pulleys or gears, evaluating their performance metrics in relation to the system's reliability. To achieve this, the components underwent rigorous testing across three distinct operational stages, during which their efficiencies were meticulously assessed using the Lagrange multiplier method. However, it was observed that the resultant efficiency values yielded real numbers that do not conform to practical, real-life applications; these outcomes are often deemed unacceptable in the context of reliability engineering. Ultimately, the study concludes that while the system reliability has been significantly enhanced, substantial adjustments were required for the critical components to express their efficiencies in integer terms. This adjustment facilitated the improvement of both component and stage reliability, taking into consideration essential factors such as cost, weight, and volume. Through this integrated approach, the author not only advances the understanding of reliability modeling in complex systems but also contributes valuable insights into optimizing component performance within the constraints of real-world operational scenarios.

This study presents a comprehensive dependability model tailored for a parallel-series configuration system, incorporating several efficiency requirements. Upon ascertaining that the data relates to real numbers, the Lagrange multiplier method is utilized to determine critical parameters such as the number of components (x_{com}), their corresponding efficiencies ($r_{Com\ Re}$), stage reliabilities ($R_{Ph\ Re}$), and overall system reliability $R_{Sys\ Re}$. The efficiencies for price, load, and size derived from this analysis are quantified as $r_{Com\ Re}$ = 0.9621, 0.9674, and 0.9714, with corresponding stage reliabilities of $R_{Ph\ Re}$ = 0.8039, 0.8745, and 0.8215. The calculated reliability of the resulting structure is $R_{Sys\ Re}$ = 0.9752. Using Lagrangean multiplier techniques, the rounding-off efficiencies are quantified as $r_{Com\ Re}$ = 0.9281, 0.9345, and 0.9457, with corresponding stage reliabilities of $R_{Ph\ Re}$ = 0.8625, 0.8745, and 0.8602. The calculated reliability of the resulting structure is $R_{Sys\ Re}$ = 0.9799. The aforementioned results yield rounded values, which are scientifically inaccurate; therefore, the author presented an alternative known as the Heuristic Algorithm technique to obtain integer solutions. The efficiencies related to price, load, and size derived from this analysis are quantified as $r_{Com\ Re}$ = 0.9687, 0.9726, and 0.9799, with corresponding stage reliabilities of $R_{Ph\ Re}$ = 0.8262, 0.8948, and 0.8675. The calculated structural reliability is $R_{Sys\ Re}$= 0.9824.

A dynamic programming approach is employed to achieve integer solutions, utilizing inputs from the Lagrange multiplier analysis for practical applicability. This yields revised component reliabilities for price, load, and size of $r_{Com\ Re}$ = 0.9722, 0.9875, and 0.9898, respectively, and stage reliabilities of $R_{Ph\ Re}$ = 0.8443, 0.9509, and 0.9308. The improved system reliability is quantified at $R_{Sys\ Re}$ = 0.9901. It is interesting that although alterations to the cost, weight, and dimensions of the components were small, these modifications resulted in substantial enhancements in stage dependability, hence augmenting system reliability at every stage and for each component. The author ultimately suggested that the dynamic programming method is superior for obtaining integer solutions, in contrast to the Lagrangean multiplier method with rounding and heuristic algorithm approaches.

The integrated reliability model (IRM) developed through this methodology proves to be exceptionally beneficial, particularly in real-world scenarios where a parallel-series configuration with reliability engineering redundancy is essential. This model offers significant benefits for design engineers specializing in dependability, particularly in scenarios where system value is constrained, as it facilitates the selection of high-quality and efficient materials. For future studies, the authors propose exploring an innovative approach that imposes constraints on the minimum and maximum reliability of components while optimizing overall system dependability. This can be achieved using contemporary heuristic methods to construct similar integrated reliability models with redundancy, thus enhancing their applicability in reliability engineering contexts.

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