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A METHOD FOR INTUITIONISTIC FUZZY MULTI-ATTRIBUTE DECISION MAKING WITH INCOMPLETE ATTRIBUTE WEIGHT INFORMATION

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ABSTRACT

A new method for solving intuitionistic fuzzy multi-attribute decision making problem is proposed, in which the information of attribute weights is incompletely known. Considering much information about hesitancy and vagueness inherited to intuitionistic fuzzy sets, a new class of distance for describing the deviation degrees between intuitionistic fuzzy sets is introduced. Furthermore, the measure of similarity degree for each alternative to ideal point is calculated by using the new proposed fuzzy distance. A model of TOPSIS is designed with the introduction of the particular closeness coefficient composed of similarity degrees for alternative ranking. Finally, a numerical example is given to show feasibility and effectiveness of the developed method.

Keywords: *Intuitionistic Fuzzy Set(IFS), Similarity Degree(SD), Closeness Coefficient(CC)*

1. INTRODUCTION

The theory of intuitionistic fuzzy set (IFS), which is the generalization of the conventional fuzzy set, was introduced by Atanassov[1,2]. The information expressed by means of the traditional fuzzy sets is not sufficient for definition of an imprecise concept. Whereas intuitionistic fuzzy set can reflect the fact that it may not always be certain that the degree of non-membership of an element in a fuzzy set is just equal to 1 minus the degree of membership in reality.

In recent years, more and more studies have been done on the theory and application of IFSs[3-8]. Szmidt and Kacprzyk proposed four distance measures between IFSs by considering the geometrical representation of the fuzzy set[3]. Wei and Zhao developed an I-IFCA and IFCA operators-based approach to solve the multiple attribute group decision making problem by using intuitionistic fuzzy sets[4]. Wei investigated the method based on the maximizing deviation method to handle the problem of MADM with incompletely known information on attribute weights to which the attribute values are given in terms of intuitionistic fuzzy numbers[5]. Li et al. developed a new methodology for solving multi-attribute group decision-making problems using IFS, in which for each decision maker in the group two auxiliary fractional programming models are derived from the TOPSIS to determine the relative closeness coefficient intervals of alternatives[6]. And optimal degrees are computed for the group to generate the ranking order of all alternatives based on the ranking method of interval number.

Hence, this paper will develop a method based on the synthesis closeness degree objectively, thus to determine the variables in decision matrix. It is expressed by means of intuitionistic fuzzy numbers. To measure the difference between two alternatives, a new fuzzy distance for intuitionistic fuzzy numbers is introduced. Then, we develop the TOPSIS approach to alternative ranking. In order to do this, this paper is arranged as follows. Section 2 firstly present some basic notations and preliminary definitions of fuzzy variables. And Section 3 introduces the presentation of the fuzzy MADM problem in which alternative values take the form of intuitionistic fuzzy numbers. And we develop a new method to measure the distances between alternatives and solve the weight values of attributes and rank alternatives. In Section 4, a

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numerical example is given to illustrate the proposed approach to the MADM problems.

2. IMPROVEMENT OF DISTANCE MEASURE

In this section, we will introduce some notations and definitions of intuitionistic fuzzy set.

Definition 1 A fuzzy set A' in $X=\{x\}$ is given by $A'=\{\langle x,\mu_{A'}(x)|x\in X\}$, where $\mu_{A'}:X\to[0,1]$ is the membership function of the fuzzy set A', $\mu_{A'}(x)\in[0,1]$ is the membership of x in A.

Definition 2 An intuitionistic fuzzy set A in X is given by $A=\{\langle x,\mu_A(x),v_A(x)\rangle|x\in X\}$ where $\mu_A:X\to[0,1]$, $v_A:X\to[0,1]$, with the condition $0\le\mu_A(x)+v_A(x)\le 1,\forall x\in X$.

The numbers $\mu_A(x),v_A(x)\in[0,1]$ denote the degree of membership and the degree of non-membership of x to A, respectively.

In this section, we will develop a new method for measuring distance between IFSs. For convenience, we express the intuitionistic fuzzy set $A=\{\langle x_i,\mu_A(x_i),v_A(x_i)\rangle|x\in X$ by $A=\langle\mu_A(x_i),v_A(x_i)\rangle$ and denote the family of all intuitionistic fuzzy sets by $IFS(X)$. Consider two intuitionistic fuzzy sets $A,B\in IFS(X)$, we propose the following distance measure between A and B:

$$d(A,B)=(\sum_{i=1}^{n}\int_{-\frac{1}{2}}^{\frac{1}{2}}\{[\frac{\mu_A(x_i)+(1-v_A(x_i))}{2}+x(1-v_A(x_i)-\mu_A(x_i))]-[\frac{\mu_B(x_i)+(1-v_B(x_i))}{2}+x(1-v_B(x_i)-\mu_B(x_i))]\}^2)^{\frac{1}{2}}dx \quad (1)$$

$$=(\sum_{i=1}^{n}\left\{\left[\frac{\mu_A(x_i)-v_A(x_i)}{2}-\frac{\mu_B(x_i)-v_B(x_i)}{2}\right]^2+\frac{1}{12}\left[(\mu_A(x_i)+v_A(x_i))-(\mu_B(x_i)+v_B(x_i))\right]^2\right\})^{\frac{1}{2}} \quad (2)$$

It can be easily checked that the distance measure $d(A,B)$ satisfies some metric properties. Firstly, $d(A,B)$ given by (2) is a nonnegative number, which is the integral of squares. It is easy to get the symmetry property $d(A,B)=d(B,A)$ from (2). In addition, the triangle inequality holds for the fact that the function to be integrated in (1) is the square of Euclidean distance.

3. INTUITIONISTIC FUZZY TOPSIS METHOD

Consider a discrete set of $n(n\ge 2)$ potential alternatives $X=\{X_1,X_2,\cdots,X_n\}$. Suppose $G=\{G_1,G_2,\cdots,G_m\}$ is the set of attributes, and $\omega=(\omega_1,\omega_2,\cdots,\omega_m)$ is the weight vector of attributes, such that ω_j ($j=1,2,\cdots,m$) is an intuitionistic fuzzy value. For the alternative $X_i\in X$ ($i=1,2,\cdots,n$) with respect to the attribute $G_j\in G$ ($j=1,2,\cdots,m$), an attribute value r_{ij} provided by the decision maker takes the form of intuitionistic fuzzy number $<\mu_{ij},v_{ij}>$. Then a fuzzy MADM problem can be concisely expressed in matrix format called intuitionistic fuzzy decision matrix $R=(r_{ij})_{n\times m}$.

Suppose $\omega_j=<\mu_j^{\omega},v_j^{\omega}>$, according to the operational law given by Atanassov[17], then the weighted attribute value is as $\omega_j\cdot r_{ij}=<\mu_j^{\omega},v_j^{\omega}>\cdot<\mu_{ij},v_{ij}>=<\mu_j^{\omega}\mu_{ij},v_j^{\omega}+v_{ij}-v_j^{\omega}v_{ij}>$ .Then the intuitionistic fuzzy weighted decision matrix is expressed as $R'=(\omega_j r_{ij})_{n\times m}$.

According to weighted decision matrix R', we extend the TOPSIS method to the intuitionistic fuzzy MADM problem. Technique for order preference by similarity to ideal solution (TOPSIS) is based upon the principle that the optimal alternative should have shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the following, we give the definitions of FPIS and FNIS.

Definition 3 For decision matrix $R=(r_{ij})_{n\times m}$, $X^+=\{r_1^+,r_2^+,\cdots,r_m^+\}$ is called the fuzzy positive ideal solution, where $r_j^+=\langle\mu_j^+,v_j^+\rangle=$

$$\begin{cases}\left\langle\max_{1\le i\le n}\omega_j\mu_{ij},\min_{1\le i\le n}\omega_j v_{ij}\right\rangle, & for\ \mathrm{j}\in\Omega_b\\ \left\langle\min_{1\le i\le n}\omega_j\mu_{ij},\max_{1\le i\le n}\omega_j v_{ij}\right\rangle, & for\ \mathrm{j}\in\Omega_c\end{cases} \quad (3)$$

and $X^-=\{r_1^-,r_2^-,\cdots,r_m^-\}$ is called the fuzzy negative ideal solution, where

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$$r_j^- = \langle \mu_j^-, v_j^- \rangle = \begin{cases} \left\langle \min_{1\le i\le n} \omega_j \mu_{ij}, \max_{1\le i\le n} \omega_j v_{ij} \right\rangle, & for\ \mathrm{j} \in \Omega_b \\ \left\langle \max_{1\le i\le n} \omega_j \mu_{ij}, \min_{1\le i\le n} \omega_j v_{ij} \right\rangle, & for\ \mathrm{j} \in \Omega_c \end{cases}$$

$$j = 1, 2, \cdots, m. \qquad (4)$$

Therefore, the measure of similarity degree for each alternative to ideal point is given by using the new proposed fuzzy distance $d(A,B)$ as $S(A,B) =$

$$1 - \frac{d(A,B)}{d(A,B) + d(A,\bar{B})} = \frac{d(A,\bar{B})}{d(A,B) + d(A,\bar{B})}, \qquad (5)$$

where $\bar{B}$ is the complementary set of B.

Obviously, the alternative which has shorter distance from the positive ideal solution and farther distance from the negative ideal solution is better than others. Then the weighted distance between each alternative X_j and the positive ideal solution is defined as

$$S_i^+ = \sum_{j=1}^{m} S(r_{ij}, r_j^+)\omega_j, \ i = 1, 2, \cdots, n,$$

Similarly, the deviation of alternative X_j to the negative ideal solution is defined as

$$S_i^- = \sum_{j=1}^{m} S(r_{ij}, r_j^-)\omega_j, \ i = 1, 2, \cdots, n.$$

With the above analysis, the weight value ω should be chosen to minimize overall distance from the positive ideal solution and maximize the distance from the negative ideal solution. To do so, we establish the following goal programming model:

$$\max \ D' = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m} S(r_{ij}, r_j^+)\omega_j}{\sum_{i=1}^{n}\sum_{j=1}^{m} S(r_{ij}, r_j^+)\omega_j + \sum_{i=1}^{n}\sum_{j=1}^{m} S(r_{ij}, r_j^-)\omega_j}$$

$$\text{s.t.} \begin{cases} \omega \in H, \omega_j \ge 0, j = 1, 2, \cdots, m \\ \sum_{j=1}^{m} \omega_j = 1 \end{cases}$$

Based on the above programming model, we can obtain the values of attribute weights. As a result, the procedure of the fuzzy TOPSIS method can be expressed in a series of steps:

step 1. Determine the positive ideal solution X^+ and negative ideal solution X^-, and measure the weighted distance respectively.

Step 2. Establish a programming model according to the normalized decision matrix $R = (r_{ij})_{n\times m}$ by applying proposed distance, so that the weight vector ω is solved based on the programming model.

Step 3. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative X_j with respect to the ideal solution is defined as

$$CC_i = \frac{S(r_{ij}, r_j^+)}{S(r_{ij}, r_j^+) + S(r_{ij}, r_j^-)}, \ i = 1, 2, \cdots, n.$$

Step 4. Ranking order of all alternatives and select the best one from a set of feasible alternatives according to the relative closeness coefficient.

4. NUMERICAL EXAMPLE

In this section, we illustrate the proposed method by considering the following numerical example. Suppose that there are four alternatives X_1, X_2, X_3 and X_4 among which decision makers have to choose. And five attributes G_1, G_2, $\cdots$, G_5 are identified as the evaluation criteria for these attributes, where the attributes G_1, G_2, G_3 and G_4 are benefit attributes, and other attribute is cost attribute. The fuzzy ratings of alternatives X_i ($i = 1, 2, \cdots, n$) according to attributes G_j ($j = 1, 2, \cdots, m$) are evaluated by decision makers and form the decision matrix $A = (a_{ij})_{n\times m}$ as listed in Table 1, and the weight information of attributes is $0.6\omega_3 \le \omega_1 \le 0.7\omega_3$, $\omega_2 \ge 0.38\omega_3$, $\omega_3 - \omega_4 \ge 0.276$, $0.2 \le \omega_5 \le 0.27$.

Table 1. Decision Matrix

	G_1	G_2	G_3	G_4	G_5
X_1	<0.10,0.09>	<0.54,0.42>	<0.18,0.35>	<0.45,0.23>	<0.11,0.80>
X_2	<0.77,0.13>	<0.62,0.24>	<0.02,0.62>	<0.33,0.10>	<0.50,0.24>
X_3	<0.47,0.44>	<0.20,0.23>	<0.58,0.22>	<0.48,0.36>	<0.68,0.19>
X_4	<0.50,0.06>	<0.63,0.31>	<0.87,0.08>	<0.52,0.13>	<0.52,0.01>

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According to the Table 1, we construct the following programming model

$$\max \ D(\omega)=$$

$$\frac{1.3389\omega_1+1.4387\omega_2+2.2835\omega_3+3.0988\omega_4+2.4932\omega_5}{4.0972\omega_1+4.4198\omega_2+4.2910\omega_3+4.8657\omega_4+4.1436\omega_5}$$

$$s.t.\ \begin{cases} 0.6\omega_3 \le \omega_1 \le 0.7\omega_3 \\ \omega_2 \ge 0.38\omega_3 \\ \omega_3-\omega_4 \ge 0.276 \\ 0.2 \le \omega_5 \le 0.27 \\ \omega_1+\omega_2+\omega_3+\omega_4=1 \end{cases}$$

By solving the above programming model, we obtain the weight values ω_j ($j=1,2,\cdots,m$) as

$CC_1=0.3345$， $CC_2=0.3274$，

$CC_3=0.6917$， $CC_4=0.6289$.

Therefore, we can rank all the alternatives X_i ($i=1,2,\cdots,n$) in accordance with the values of d_j as $X_3 \succ X_4 \succ X_1 \succ X_2$, and then the best alternative is X_3 .

5. CONCLUSIONS

In this paper, we have investigated the method for the MADM problem, in which the preference values take the form of intuitionistic fuzzy numbers, and the values of attribute weights are partly known. A new fuzzy distance for intuitionistic fuzzy numbers is introduced. By using new distance between fuzzy variables, we have established a goal programming model from which a simple and exact formula is derived to determine the attribute weights. Then TOPSIS method has been extended for ranking alternatives, and a numerical example has been given to illustrate the developed approach. The numerical example shows that the proposed method in this paper is simple and effective.

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