

PROFIT MODEL OF GROUP-BUY WEBSITES BASED ON PRINCIPAL-AGENT THEORY

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ABSTRACT

The future profit model of group-buy websites and incentive problem of websites to suppliers is studied using principal-agent theory which mainly analyzes the group-buy websites in supply chain. And a conclusion can be drawn that the stronger capacity of the group-buy websites, the more struggling level they will pay. At the same time, the stronger capacity of the sellers, the smaller struggling cost coefficient, and the stronger sellers can get more excitation of royalty rate.

Keywords: *Group-buy Website, Profit Model, Principal-agent, Incentive Mechanism*

1. INTRODUCTION

With the rapid development of group-buy websites, the network operation also appears a lot of problems. Such as the profit model of group-buy websites, low credit, uneven product quality, and so on. These problems have gradually become the focus of academic research. In 2002, Kauffman defined group-buy websites as: Online group shopping refers to combing the consumers who have the willingness to improve the bargaining power of suppliers using the network, thus to buy goods at a low price[1]. Group-buy websites combine sellers with group-buy alliance, and play the price game among the group-buy alliance in order to achieve the maximization of their own interests. During its process of integrating the upstream supply resources in supply chain, because of the existence of information asymmetry, suppliers need some mechanism to encourage group-buy websites to choose the actions in favor of themselves and then achieve better resources integration. Therefore, the research to group-buy websites profit model – incentive scheme has very important significance. Krishan S.Anand and Ravi Aron analyzed the basis of the existence of group-buy websites from dynamic pricing mechanism and have studied relevant models[2]; With the consumer's view, Zhao Baoguo studied the quantity price discounts and has established a virtual group-buy website model[3]. The above documents analyzed group-buy websites from different

aspects. Some scholars found that the relationship between group-buy websites and the suppliers can be solved by the principal-agent theory and have made some research. Liao Kaiji and other scholars analyzed the various strategy choices among the websites, suppliers and consumers based on the hypothesis of limited rationality and evolutionary game theory. And they further analyzed the influence to the economic market of each party after long-term study and strategy adjustment [4]. Deng Anping and others went on to study the profit model of group-buy websites under asymmetric information and analyzed the incentive problems to group-buy websites offered by the sellers [5].

This paper, however, differs from those studies above in this aspects: it analyzes the profit model of group-buying websites on the base of principal-agent relations. And it gives dominance to these websites in supply chains and takes the consideration of Game analysis between websites and vendors on condition of asymmetric information. Next a optimized model should be set up and solved. Then an analysis should be given to the solution. Lastly the major conclusion will be verified via factual examples.

2. THE PRINCIPAL-AGENT MODEL OF GROUP-BUY WEBSITE

2.1 Model Assumption

The following assumptions are made according to principal-agent theory and group-buy websites features:



①Supposing group-buy websites cannot observe the effort level of sellers, but their output can be calculated.

②The literatures [6,7,8,9] make the hypothesis that the output function is linear function with a constant output coefficient. The paper [10] assumes that the output coefficient is related to effort level. According to the relations between group-buy websites and the principal-agent, the output coefficient will be affected by the effort level of group-buy websites and is not constant. Therefore, according to the linear output function set in literature[8,10]and combing with the characteristics of the sellers, we assume the output function as follows:

$$\pi = f(x_w)x_s + \xi, \xi \sim N(0, \sigma^2) \quad (1)$$

where ξ is a random error, indicating external uncontrollable factors. x_w shows the effort level of group-buy websites which relate with human and material resources cost of group-buy websites spending on information sharing, training, quality guidance, coordination and management. x_s is the effort level of suppliers, and can be measured by the size of comprehensive weighted average cost of human and material resources spending to improve the ability of suppliers and to increase output quantity. $f(x_w)$ is the output coefficient of the sellers, which is effected by the effort level of group-buy websites, and it is a function on the effort level x_w . According to the principles of microeconomics, $f(x_w)$ satisfies the law of diminishing marginal returns.

③The literatures[6,7, 9, 11]assume that the client is risk neutral and the agent to be risk aversion. But in reality group-buy websites and the suppliers are both rational economic man. Therefore, according to the literatures [6,8,10,11], combing the feature of rational economic man, assuming that group-buy websites and the suppliers are risk aversion, considering the risk aversion is constant, we assume its utility function as follows:

$$U = -e^{-\rho\omega} \quad (2)$$

where ρ is the absolute risk aversion; ω represents the real money income.

④ According to the basic principle of microcosmic economics and combined with the literatures [8,10,11], we assume the equivalent effort cost function of group-buy websites is $C(w) = a_w x_w^2 / 2$. a_w is the effort cost coefficient of group-buy websites. x_w is related to the ability of group-buy website, the stronger capacity, the

smaller x_w . Similarly, the equivalent effort cost function of suppliers is $C(s) = a_s x_s^2 / 2$. a_s represents the effort cost coefficient, showing a negative correlation with the capacity level of itself.

⑤To encourage the sellers to work hard and reduce their cost, suppose that the group-buy websites and the suppliers sign the contract:

$$R(\pi) = R_0 + b\pi \quad (3)$$

In this contract, $R(\pi)$ is the reward to the merchant by group-buy websites, R_0 represents the fixed payment and b is the incentive coefficient, that is, the royalty percentage.

2.2 Modeling

According to the above and the assumption ③ that both the group-buy websites and the sellers are risk aversion and considering that the risk aversion is invariant, make the risk aversion quantity of them are ρ_s and ρ_w , then, the utility functions of group-buy websites and the sellers, respectively, are as follows:

$$U(w) = -e^{-\rho_w \omega_w} \quad (4)$$

$$U(s) = -e^{-\rho_s \omega_s} \quad (5)$$

ω_w , ω_s represent the real yields of the group-buy websites and the sellers.

According to Arrow-Pratt conclusion [1], the risk cost of the group-buy websites and the sellers are as follows respectively:

$$C(\rho_w) = \rho_w b^2 \sigma^2 / 2 \quad (6)$$

$$C(\rho_s) = \rho_s b^2 \sigma^2 / 2 \quad (7)$$

The real yields of the group-buy and the sellers are as follows respectively:

$$\omega_w = \pi - R(\pi) - C(w) = (1-b)\pi - R_0 - a_w x_w^2 / 2 \quad (8)$$

$$\omega_s = R(\pi) - C(s) = R_0 + b\pi - a_s x_s^2 / 2 \quad (9)$$

The certainty equivalence revenue of the sellers is:

$$E\omega_s - C(\rho_s) = R_0 + b\pi - \frac{1}{2} a_s x_s^2 - \frac{1}{2} \rho_s b^2 \sigma^2 \quad (10)$$

In the same way, the certainty equivalence revenue of the group-buy websites is as follows:

$$E\omega_w - C(\rho_w) = (1-b)\pi - R_0 - \frac{1}{2} a_w x_w^2 - \frac{1}{2} \rho_w b^2 \sigma^2 \quad (11)$$

Due to that the maximum expected utility function $EU_s = -Ee^{-\rho_s \omega_s}$ is equivalent to the maximum certainty revenue, therefore, we use the certain income replace the expected utility. The retained income level of the sellers is ω_s^0 , when the certainty equivalence revenue is less than ω_s^0 , the



sellers do not accept the contract. So the constraint on the sellers involved in cooperation is:

$$IR \quad R_0 + b\pi - \frac{1}{2}\alpha_s x_s^2 - \frac{1}{2}\rho_s b^2 \sigma^2 \geq \omega_s^0 \quad (12)$$

In case that the group-buy websites have given the rewards to the sellers, the sellers will choose the optimal effort level to maximize their certainty equivalence revenue. Therefore, for the sellers the incentive compatibility constraint conditions following must also be met:

$$IC \quad \text{Max}_{x_s} \left\{ R_0 + b\pi - \frac{1}{2}\alpha_s x_s^2 - \frac{1}{2}\rho_s b^2 \sigma^2 \right\} \quad (13)$$

In view of the asymmetric information, group-buy websites cannot observe the effort level of the suppliers. Therefore, they must realize the expected profit maximization while incenting the suppliers to work hard. Based on the above analysis and assumptions, we establish the following principal-agent model:

$$O.B. \text{Max} \left\{ (1-b)\pi - R_0 - \frac{1}{2}\alpha_w x_w^2 - \frac{1}{2}\rho_w b^2 \sigma^2 \right\} \quad (14)$$

$$(IR) \quad R_0 + b\pi - \frac{1}{2}\alpha_s x_s^2 - \frac{1}{2}\rho_s b^2 \sigma^2 \geq \omega_s^0 \quad (15)$$

$$(IC) \quad \text{Max} \left\{ R_0 + b\pi - \frac{1}{2}\alpha_s x_s^2 - \frac{1}{2}\rho_s b^2 \sigma^2 \right\} \quad (16)$$

Supposing that the sellers have no difference in the acceptance of a contract, and then the sellers always choose to accept the contract. And rational group-buy websites do not give the sellers more pay. So, in most cases, the equation (2) participated in the constraint is true, combined with the papers [12] and [13], the following can be gained through solving:

$$b = \frac{f^2(x_w)}{f^2(x_w) + \alpha_s \rho_s \sigma^2 + \alpha_w \rho_w \sigma^2} \quad (17)$$

$$R_0 = \omega_s^0 + \frac{\alpha_s \rho_s \sigma^2 f^4(x_w) - f^6(x_w)}{2\alpha_s [f^2(x_w) + \alpha_s \sigma^2 (\rho_s + \rho_w)]^2} \quad (18)$$

3. DISCUSSION OF RESULTS

Based on the principal-agent theory, the problem for group-buy websites to give the sellers reasonable percentage (incentive coefficient) need further analysis to model:

① From the (17) equation, the value of royalty ratio b of sellers and $f^2(x_w)$ have a positive correlation. The higher level of the group-buy websites, the greater royalty rate the sellers can get.

② From the (17) equation, the value of royalty ratio b of sellers is negatively correlated to the risk aversion ρ_w of group-buy websites. The greater ρ_w ,

the more afraid of risk the group-buy websites are, and the royalty rate that sellers can get is smaller.

When $\rho_w \rightarrow 0$, which means that group-buy websites risk is neutral, $b = \frac{f^2(x_w)}{f^2(x_w) + \alpha_s \rho_s \sigma^2}$ can be interpreted that the sellers' royalty rate b is not affected by the risk influence of the sellers; when $\rho_w \rightarrow \infty$, that is, the risk aversion is infinite, then the royalty rate is 0.

③ From the (17) equation, the value of b is passively correlated to the risk aversion of group-buy websites. The bigger risk aversion degree, the more afraid of risk, and the royalty rate given by group-buy websites is smaller.

④ From the (17) equation, the sellers' royalty rate b has a negatively correlation to the sellers' effort cost coefficient α_s . The bigger α_s , the weaker ability of the sellers, and the weaker sellers get smaller royalty rate from group-buy websites.

⑤ From the (17) equation, the value of b is negatively correlated to the exogenous uncertainty factor σ . The more exogenous uncertainty factors, the more unstable the social economic conditions, and group-buy websites give sellers smaller royalty rate. When $\sigma=0$, $b=1$ then there is no exogenous factors, the royalty ratio is 1, and this ideal state in reality does not exist. When $\sigma \rightarrow \infty$, the exogenous uncertainty is too large, and group-buy websites do not give incentive coefficient, only a fixed payment.

4. EXAMPLE ANALYSIS

To further illustrate how group-buy websites determine the optimal remuneration given to suppliers and their own efforts in different circumstances, here we give several numerical examples in specific cases. According to the assumption ②, the function relations between the sellers' output coefficient and the effort level of group-buy websites is set as follows:

$$f(x_w) = \sqrt{x_w} \quad (19)$$

Plug equation (6) into (1) (4) (5) and get:

$$b = \frac{x_w}{x_w + \alpha_s (\rho_s + \rho_w) \sigma^2} \quad (20)$$

$$R_0 = \omega_s^0 + \frac{\alpha_s \rho_s \sigma^2 x_w^2 - x_w^3}{2\alpha_s [x_w + \alpha_s \sigma^2 (\rho_s + \rho_w)]^2} \quad (21)$$

$$\text{Max} \left\{ (1-b)\pi - R_0 - \frac{1}{2}\alpha_w x_w^2 - \frac{1}{2}\rho_w b^2 \sigma^2 \right\} \quad (22)$$



Case1: Determination of the optimal royalty rate b when the sellers' effort cost coefficient α_s is different.

If $\sigma=10$, $\rho_w=4$, $\rho_s=2$, $x_w=128$, $\omega_s^0=108$, put the parameters into the equation (20) (21) (22). When $\alpha_s=0.4$, $R_0=100.74$, $b=34.78\%$. When $\alpha_s=0.8$, $R_0=108.89$, $b=21.05\% < b=34.78\%$. Visibly, when the sellers' effort cost coefficient increases, the ability of the sellers get weaker, then group-buy websites will give a smaller royalty rate. The fixed payments by group-buy websites respectively are 100.74 and 108.89.

Case2: Determination of the optimal royalty rate when the exogenous uncertainty factor is changed.

If $\rho_w=4$, $\rho_s=2$, $x_w=128$, $\alpha_s=0.8$, $\omega_s^0=108$, put them into the equation (20) (21) (22) respectively. When $\sigma=10$, $R_0=108.89$, $b=21.05\%$. When $\sigma=12$, $R_0=110.11$, $b=15.63\% < b=21.05\%$. We can see that the bigger exogenous uncertainty σ , the more uncertainty factors the sellers' external environment has, and the sellers get smaller royalty rate. The fixed pays are 108.89 and 110.11 respectively.

Case3: Determination of the optimal royalty rate when the sellers' risk aversion is different

If $\sigma=10$, $\rho_w=4$, $x_w=128$, $\alpha_s=0.8$, $\omega_s^0=108$, take them into formula (20)(21)(22). When $\rho_s=2$, $R_0=108.89$, $b=21.05\%$. When $\rho_s=4$, $R_0=111.33$, $b=16.67\% < b=21.05\%$. From the above, when the risk aversion increases, the sellers are more unwilling to take risks and more afraid of risk, and group-buy websites will give it a smaller royalty rate. The optimal fixed payments are 108.89, 111.33 respectively.

Case4: Determination of the optimal royalty rate when the group-buy websites' risk aversion is different

If $\sigma=10$, $\rho_s=2$, $x_w=128$, $\alpha_s=0.8$, $\omega_s^0=108$, take into (20) (21) (22). When $\rho_w=2$, $R_0=109.63$, $b=28.57\%$. When $\rho_w=4$, $R_0=108.89$, $b=21.05\% < b=28.57\%$. Visibly, when the group-buy websites' risk aversion increases, illustrating that group-buy websites are more afraid of risk, then they give sellers a smaller royalty rate. The optimal fixed payments are 109.63, 108.89 respectively.

Case5: Determination of the group-buy websites' optimal effort level

Shift the equation (20) and we can get:

$$x_w = \frac{b\alpha_s(\rho_s + \rho_w)\sigma^2}{1-b} \quad (10)$$

If $\sigma=10$, $\rho_s=2$, $\rho_w=4$, $\alpha_s=0.8$, $b=0.15$, $\omega_s^0=108$, put the parameters into (23) respectively, then $x_w=84.71$. We can see that in the case of given parameters the optical effort level is 84.71.

5. CONCLUSION

Above all, according to the principal-agent relations between group-buying sites and suppliers, regarding those sites as a dominant role in supply chains, we concern the impact on suppliers' effort output coefficient by the level of effort made by the sites. And a further study is carried out on the profit model of the group-buying sites; besides, we perform an analysis on suppliers' optimized proportion of percentage and draw a conclusion illustrated as follows: the more capable the group-buying sites is of themselves, the higher the level of effort is, and the larger proportion of vendors' percentage is; the greater the degree of risky aversion of the sites and vendors, the less the proportion given to vendors is; the more capable vendors are, the smaller the value of the effort-cost coefficient is and so the greater the incentive given by group-buying sites. In the last part, we perform an analysis on factual examples and further explain the optimized proportion of percentage given to vendors, the optimized fixed payment and the ways to identify the optimized level of effort. In this article, it is based on the one-to-one principal-agent relations between group-buying sites and suppliers, irrespective of the cases in which one site responds to multiple vendors. Though, this one-to-many incentive mechanism is our next research goal.

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