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RELIABILITY ESTIMATION OF PRODUCTS USING TIME SERIES ANALYSIS AND GREY THEORY BASED ON STEP-STRESS ADT

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ABSTRACT

This paper proposes a SSADT data analysis method to estimate product reliability based on time series analysis and grey theory. A RTVAR degradation time series model is proposed based on SSADT data. A grey prediction method of RTVAR model is put forward to predict product lifetime. By using the suggested method, product reliability is obtained. An example is presented as a verification of the modeling technique and estimation method. A reasonable estimation of lifetime and reliability of the product is obtained by employing the present method.

Keywords: *Reliability Estimation, Time Series, SSADT, Degradation Data, Grey Theory*

1. INTRODUCTION

For long lifetime and high reliability products, it is difficult to obtain failure time data in a short time period. Hence, Accelerated Degradation Testing (ADT) is presented to deal with the cases that few or no failure time data could be obtained but degradation data of the primary parameter of the product are available. Step-Stress ADT (SSADT) is commonly used for the advantage that it needs only a few test samples to conduct a life test. For reliability and lifetime evaluation in SSADT, previous works use deterministic functions to represent the product performance degradation process. However, it does not represent performance degradation information adequately. It is necessary to add stochastic information description to performance degradation process.

Time series analysis can represent stochastic information. During the last two decades, considerable research has been carried out in time series analysis. However, only few papers have studied the degradation data analyze method based on time series method. Moreover, SSADT data analysis based on time series method has not been reported in literature at present.

Product performance degradation data of SSADT is time series. Hence, a time series method is proposed to analyses SSADT data. There are several problems for applying time series method to SSADT data analysis as follows.

1 Product lifetime can be obtained by converting SSADT data at each test stress level into use-stress level. According to section 2 in this paper, this converted data is usually unequally spaced time series. Using traditional time series analysis method, which is only suitable for equally spaced series, might cause big prediction error.
2 Traditional degradation modeling method only considers degradation process of product performance. However, certain control characteristic of accelerated test equipment also causes product degradation.
3 Product lifetime is predicted based on modeling SSADT time series. However, accurately prediction of time series model depends on large sample size. Traditional direct prediction is not effective enough to time series model.

In this paper, four assumptions of SSADT are put forward:

1 Product performance level degrades monotonously.
2 The failure mechanism is unchanged at all stress levels.
3 The remaining life of specimens depends only on the current cumulative fraction failed and current stress.
4 There is no failure unrelated to degradation failure mechanism.

There are certain notations of SSADT in this paper:

k is number of the stress levels;

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S_i is i^{th} stress level, $i = 1,2,\cdots,k$;

Δt is sampling interval;

m_i is number of measurements under S_i ;

m is totally number of measurements, $m = \sum_{i=1}^{k} m_i$;

τ_i is the time scale of S_i , $\tau_i = \Delta t \cdot m_i$

2. SSADT DATA CONVERSION

For modeling SSADT data at use-stress level, it is necessary to convert SSADT data at each test stress level into use-stress level. A conversion method is presented based on stress-degradation rate relationship and cumulative exposure model.

2.1 Stress-Degradation Rate Relationship

Cumulative degradation measure at S_i is obtained from the preprocessed SSADT data using the equation

$$\Delta y_i = y_{m_i} - y_{0_i} \tag{1}$$

Here, Δy_i is cumulative degradation measure at S_i , y_{0_i} is the first data at S_i .

Based on Eq.1 and τ_i , the degradation rate slp_i at S_i is obtained by the equation

$$slp_i = \frac{\Delta y_i}{\tau_i} \tag{2}$$

In SSADT, there are different degradation rates versus different stress levels. Certain stress-degradation rate relationship can be used to represent relationship between degradation rate and stress. Thus, degradation rate slp_0 at use-stress S_0 can be obtained.

2.2 Cumulative Exposure Model

Time scale τ_i is converted into time scale τ_{0i} at S_0 based on cumulative exposure model. The equation is

$$\tau_{0i} = \frac{\Delta y_i}{slp_0} = \frac{slp_i}{slp_0} \cdot \tau_i \tag{3}$$

Sampling interval Δt is converted by equation

$$\Delta t_i = \frac{\tau_{0i}}{m_i} = \frac{slp_i}{slp_0} \cdot \frac{\tau_i}{m_i} = \frac{slp_i}{slp_0} \cdot \Delta t \tag{4}$$

Here, Δt_i is sampling interval at S_0 .

Δt_i is different each other. Hence, the converted SSADT data at S_0 is unequally spaced data.

3. TIME SERIES MODELING

Product lifetime is obtained by modeling SSADT time series at S_0 , which is unequally spaced and nonstationary time series. A new time series modeling method is put forward to represent the SSADT data at S_0 .

In this paper, $Y(l)$ denotes the level of degradation at l^{th} measurements, $l = 1,2,\cdots m$. According to the Cramer Decomposition Theorem, any time series $Y(l)$ can be decomposed into deterministic component and stationary random component. $T(l)$ and $S(l)$ denotes trend component and seasonal component, which are deterministic components. $R(l)$ denotes residual stationary random component.

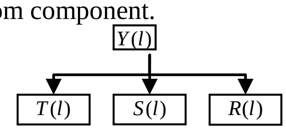

Figure1 Relationship of components

3.1 Trend Component at S_i Modeling

Based on preprocessed SSADT data, trend component at S_i , denoted by $T_i'(l)$, is different each other and represented respectively by linear regression model

$$T_i'(l) = slp_i \cdot l + y_{0_i} \tag{5}$$

3.2 Seasonal Component Modeling

Controlled by test equipment, product performance reflects periodical fluctuation of stress level. This paper regards it as seasonal component $S(l)$ and represents it using Hidden Periodicity (HP) regression model

$$S(l) = \sum_{j=1}^{q} A_j \cos(\omega_j l + \varphi_j) \tag{6}$$

Here, q is number of angular frequency, ω_j is j^{th} angular frequency, A_j is amplitude of ω_j , φ_j is j^{th} phase angle.

3.3 Residual Component Modeling

$R(l)$ represents residual series at S_0 . This unequally spaced stationary random series is time series with stationary correlation coefficient [1]. t_l denotes the l^{th} sampling time at S_0 , this paper represents $R(l)$ using Time Varying AutoRegressive (TVAR) model

$$R(l) = \sum_{j=1}^{p} \eta_j(\tau_l) R(l-j) + \varepsilon(\tau_l) \tag{7}$$

Here,

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$\tau_l = (\tau_{l1}, \tau_{l2}, \cdots, \tau_{lp})', \tau_{lj} = t_l - t_{l-j}, j = 1,2,\cdots, p$, $\varepsilon(\tau_l)$ obeys $N[0, \sigma_0^2 \phi(\alpha, \tau_l)]$, it is independent white noise.

3.5 Trend Component at S_0 Modeling

Trend component at S_0 denoted by $T(l)$ is represented by

$$T(l) = slp_0 \cdot l + y_{0_1} \tag{8}$$

3.6 Degradation Time Series Modeling

This paper proposes degradation time series model, Regression Time Varying AutoRegressive (RTVAR) model

$$Y(l) = T(l) + S(l) + R(l) \tag{9}$$

4. RELIABILITY ESTIMATION

In ADT, failure occurs as product performance level achieves a specified threshold. Product lifetime is time scale from ADT beginning to the first achieving. In this paper, the lifetime is obtained by prediction of degradation model. This predicted lifetime is called pseudo lifetime.

In addition, the accuracy of pseudo lifetime depends on prediction precision of degradation model. The creditability of reliability estimation depends on accuracy of pseudo lifetime.

4.1 Pseudo Lifetime

Pseudo lifetime t_{life} is obtained when prediction data y_{m+u} first achieves the failure threshold at prediction step u_{life}, it is obtained by equation

$$t_{life} = \sum_{i=1}^{k} \tau_{0i} + u_{life} \cdot \overline{\Delta t} \tag{10}$$

4.2 Prediction Error

Prediction error of time series model is brought by residual component. Its mean square error of grey prediction is obtained by u_{life} and equation

$$\sigma_{m+u}^2 = \sum_{j=1}^{u} G_{uj}^2 \sigma_{(u-j+1)\varepsilon}^2 \tag{11}$$

Here, G_{uj} is the Green function of TVAR model u-step prediction, $\sigma_{(u-j+1)\varepsilon}^2$ is white noise variance of TVAR model $(u-j+1)$-step prediction, $j = 1,2,\cdots,u$.

4.3 Reliability Estimation

Pseudo lifetime distribution and reliability estimation are obtained based on all product pseudo lifetime.

5. GREY PREDICTION

Prediction precision of time series model depends on sample size. Hence, accurate prediction of time series model needs plenty of samples. It is difficult to put into practice. Grey theory is a prediction method based on data with small sample size and indeterminacy. This paper puts forward a grey lifetime prediction method.

The system grey prediction nesting method of grey theory nests GM(1,1) in GM(1,N) to obtain prediction values based on prediction model. According to it, the first data is removed from a series and a one-step prediction data is added in this series. Modeling this new series can update model parameters. Repeat this procedure until a set step. Prediction precision of this nesting prediction is higher than of direct prediction.

This paper predicts RTVAR model using the system grey prediction nesting method. There is prediction procedure of SSADT data at S_0 as follows. Fig.2 shows flow chart of the prediction procedure.

Step1: h is set as the number of prediction times. u denotes prediction times, $u = 1,2,\cdots,h$.

Step2: $\{y_l\}$ denotes a not-predicted preprocessed SSADT series.

Step3: Parameters of Eq.5~8 are updated at u^{th} prediction.

Step4: Set $\overline{\Delta t}$ as prediction interval, $\overline{\Delta t} = \sum_{i=1}^{k} \Delta t_i$ then $\tau = (\overline{\Delta t}, \overline{\Delta t} \cdot 2, \cdots, \overline{\Delta t} \cdot p)'$.

A one-step prediction data y^* is obtained by the optimal unbiased prediction model of Eq.9. Set $y_{m+u} = y^*$.

Step5: y_1 is removed from $\{y_l\}$ and y^* is added in it, a new series $\{y_l, y^*\}$ is obtained.

Step6: $\{y_l, y^*\}$ is regarded as the not-predicted series, Step 3~4 are repeated.

Step7: Step 5 and 6 are repeated $h-1$ times, y_{m+u} is obtained one by one.

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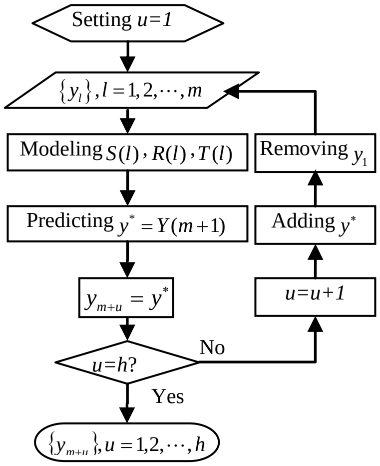

Figure 2: Flow chart of prediction procedure

6. EXAMPLE VERIFICATION

A four temperature levels SSADT of 4 certain products is conducted as an example to verify the suggest SSADT data analysis method. Sampling interval is one hour. Fig.3 shows the original degradation data of SSADT. Tab.1 shows test parameters.

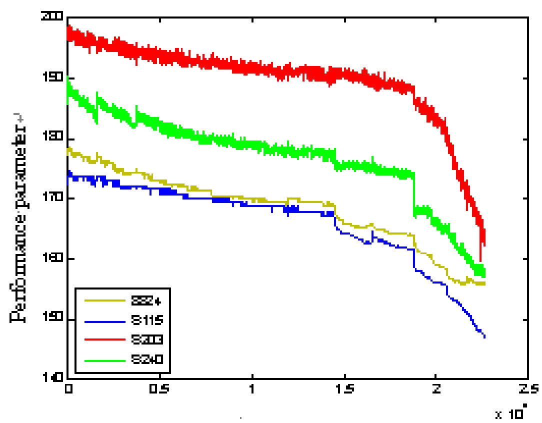

Figure 3 Original SSADT data of 4 products

Table 1 Parameters of SSADT

Temperature	Sample size
60 ℃	14519
80 ℃	4292
100 ℃	1806
110 ℃	2052

Firstly, SSADT data of each product is preprocessed by initial value processing for eliminating influence of its initial value difference and normalizing the failure criterion. Fig.4 shows the preprocessed SSADT data.

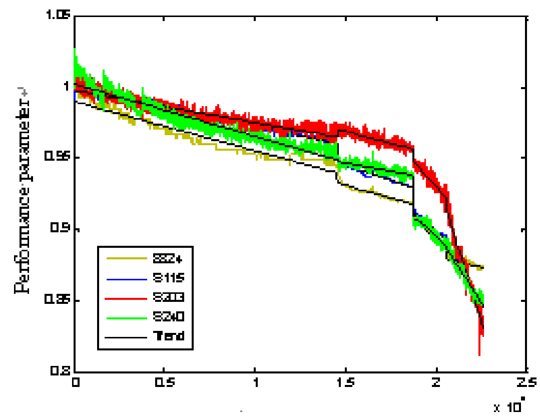

Figure 4 Preprocessed SSADT data of 4 products

Secondly, SSADT data at each temperature are converted into it at use-stress level 25℃. Fig.5 shows the Arrhenius temperature stress-degradation rate relationship.

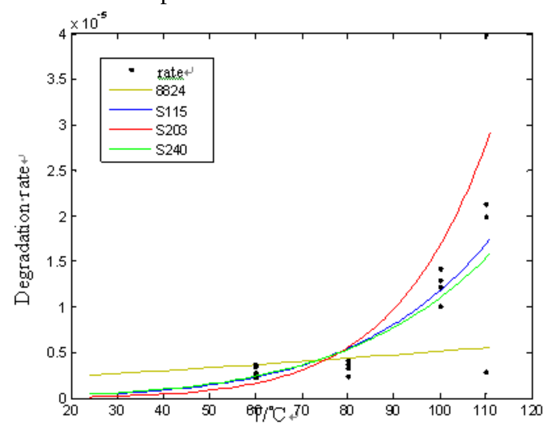

Figure 5 Stress-degradation rate relationships of 4 products

Thirdly, the unequally spaced preprocessed SSADT time series of each product at 25℃ is represented by RTVAR model. Parameters of the TVAR model are estimated by the maximum likelihood method.

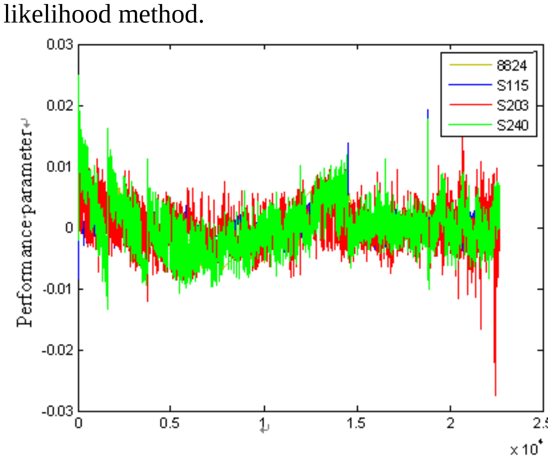

Figure 6 Residual component data of 4 products

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Fourthly, SSADT prediction data of each product is obtained. Fig.7 shows the SSADT prediction data. Black line shows prediction data.

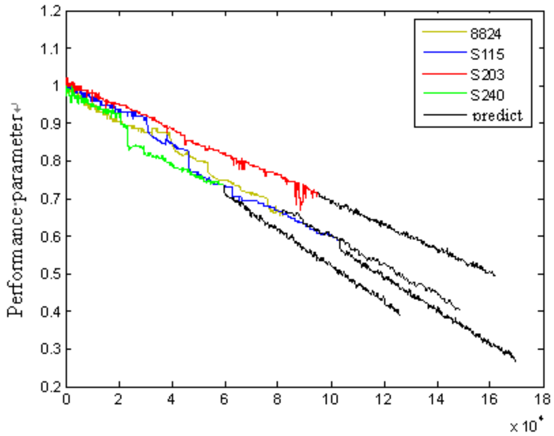

Figure 7 SSADT prediction data of 4 products

Fifthly, set 97% of initial value as failure threshold, Tab.2 shows pseudo lifetime and prediction error. Set lognormal distribution as pseudo lifetime distribution. Fig.8 shows reliability function. Tab.3 shows pseudo lifetime distribution parameters estimated using maximum likelihood method.

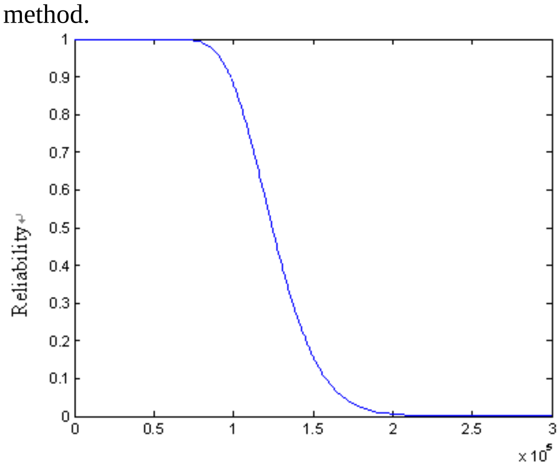

Figure 8 Reliability function

Table 2 Pseudo lifetime & prediction error of SSADT

Sample	Pseudo Lifetime (hours)	Mean square error
8824	124100	0.0053
S115	117767	0.003
S203	159986	0.0026
S240	102335	0.0034

Table 3 Parameters of Pseudo lifetime distribution

Model	RTVAR
Log-lifetime mean	11.8316
Log-lifetime variance	0.0348
Median lifetime (hours)	137448

7. CONCLUSION

This paper proposes a SSADT data analysis method based on time series analysis and grey theory. A RTVAR degradation time series model is proposed based on SSADT data. A grey prediction method of RTVAR model is put forward to predict product lifetime. By using the suggested method, product reliability is obtained. An example is presented as a verification of the modeling technique and estimation method. A reasonable estimation of lifetime and reliability of the product is obtained by employing the present method.

There are several advantages of this suggested SSADT data analysis method based on time series. First, it solves the problem of SSADT data analysis by using TVAR model based on unequally spaced time series with stationary correlation coefficient. Second, it represents influence of test equipment by using HP regression model. Third, RTVAR model merges both virtues of regression model and TVAR model to improve model precision and enhance creditability of lifetime prediction and reliability estimation. Fourth, grey prediction of RTVAR model preserves virtue and overcome weakness of time series analysis which prediction precision depends on sample size.

There are also several problems about SSADT data analysis based on time series analyses deserve further research. Indeed, the way that the improvement of prediction precision of time series model enhances creditability of lifetime prediction and reliability estimation in SSADT is not clarified. Thus, it can be the next object of research to improve reliability estimation technology further.

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