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THE ADAPTIVE OUTPUT SYNCHRONIZATION OF DIFFERENT-ORDER CHAOTIC SYSTEMS

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ABSTRACT

In this paper, the synchronization of the outputs for the uncertain chaotic systems with different-orders is investigated. The adaptive control strategy is applied to make output of the slave system track those of the master, despite the chaotic systems with different-orders and uncertainties. The proposed controller can ensure synchronous error converges to zero. Simulation results confirm the effectiveness of the proposed method.

Keywords: *Synchronization, output-Synchronization, adaptive control, chaotic system*

1. INTRODUCTION

Since Pecora et al [1] proved the possibility of controlling chaos in early 90', the chaos synchronization has received noticeably attention due to its potential applications such as secure communication, biological systems, digital communication, chemical reaction and design, and so on. For the chaotic systems synchronization, there were several methods proposed, for example: complete (or anti) synchronization, phase synchronization, lag synchronization, generalized synchronization, intermittent lag synchronization, modified function projective synchronization [2]-[16].

Uncertainty (or disturbance) of the system is the common in real world. The system parameters are not constant when the system works. The uncertainty (or disturbance) may degrade performance of the system. To the best of our knowledge, very few have been studied the varying parameters. Wang et al. investigated the anti-synchronization in non-identical chaotic systems with known or unknown parameters [3]. Lag synchronization of a class of chaotic systems with unknown parameters was proposed in [6]. Tao et al presented the time-delayed generalized projective synchronization of piecewise chaotic system with unknown parameters [14].

In this paper, we propose a new synchronization scheme, i.e., output-synchronization of the chaotic systems with different-order and uncertainties. An adaptive controller is formulated. A sufficient condition is proposed to guarantee synchronous error dynamics stable. Finally, an example is demonstrated to verify the effectiveness of the criterion proposed.

2. PROBLEM STATEMENT

Consider the master system as below:

$$\dot{x} = f(x) + \Delta f(x,\sigma,t), \tag{1}$$

$$z_1 = C_1 x, \tag{2}$$

and the slave system is given by

$$\dot{y} = g(y) + \Delta g(y,\upsilon,t) + Du, \tag{3}$$

$$z_2 = C_2 y. \tag{4}$$

In the above equations, $x \in R^n, y \in R^m$ are states, $u \in R^p$ is input and $z_1, z_2 \in R^p$ are outputs $(1 \le p \le \min\{n,m\})$. vector functions $f(\,) \in R^n$, $g(\,) \in R^m$ and $\Delta f(\,) \in R^n$, $\Delta g(\,) \in R^m$ are known nonlinear terms and the nonlinear uncertainty experienced by the systems. It is assumed that all the nonlinear functions are smooth enough. σ and υ are parameter disturbances, and satisfying $\|\sigma\| \le \delta_1, \|\upsilon\| \le \delta_2$, δ_1 and δ_2 are positive constants . $D \in R^{n\times p}$ is full column rank constant matrix , $C_1 \in R^{p\times n}$ and $C_2 \in R^{p\times m}$ are full row rank ones.

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The objective of this work is to synchronize the outputs of the different-order master-slave chaotic systems.

Assumption 1. The uncertainty terms

$C_1\Delta f(x,\sigma,t)=\xi(x,\sigma,t)=[\xi_1,\dots,\xi_n]^T$,

$C_2\Delta g(y,\upsilon,t)=\zeta(y,\upsilon,t)=[\zeta_1,\dots,\zeta_m]^T$ are assumed to be bounded. Therefore, there exists ρ_{ji} such that

$$|\xi_i|\le\rho_{1i},i=1,2,\dots,n \tag{5}$$

$$|\zeta_i|\le\rho_{2i},i=1,2,\dots,m \tag{6}$$

where $\rho_{ji}>0$ is constant, $j=1,2$.

3. THE MAIN RESULTS

The synchronization error of the outputs for the systems (1) and (2) is defined as $e=z_1-z_2$, i.e.

$$e=C_2y-C_1x. \tag{7}$$

By derivative of both sides of (7), we obtain

$$\dot e=C_2\dot y-C_1\dot x$$
$$=C_2(g(y)+\Delta g(y,\upsilon,t)+Du)$$
$$-C_1(f(x)+\Delta f(x,\sigma,t)).$$

Then the error dynamics is given by

$$\dot e=F+\Delta F+C_2Du. \tag{8}$$

Where $F=C_2g(y)-C_1f(x)$,

$$\Delta F=[\Delta F_1,\dots,\Delta F_p]^T$$
$$=C_2\Delta g(y,\upsilon,t)-C_1\Delta f(x,\sigma,t)$$
$$=\xi-\zeta.$$

It is easy to see that

$$|\Delta F_i|\le\rho_i=\rho_{1i}+\rho_{2i}. \tag{9}$$

The adaptive continuous control law is proposed as:

$$u=-(C_2D)^{-1}(F+u_\Delta+u_k), \tag{10}$$

where

$u_\Delta=[u_{\Delta1},\dots,u_{\Delta p}],u_k=[u_{k1},\dots,u_{kp}]^T$,

$u_{ki}=k_ie_i$,

$u_{\Delta i}=\hat\rho_i\tanh(\theta_ie_i)$, $i=1,2,\dots,p$.

$\hat\rho_i$ is the estimate of the unknown constant ρ_i .The suitable adaptive law is defined as:

$$\dot{\hat\rho}_i=-|e_i|,i=1,2,\dots,p \tag{11}$$

Theorem 1. Consider the error system (8), With the control law u in Eq.(10) and adaptive law in Eq.(11), then, the error e in (7) converges to zero.

Proof: Consider the following Lyapunov function:

$$V_1=\frac{1}{2}e^Te. \tag{12}$$

Taking derivative of both sides of (12) with respective to time yields:

$$\dot V_1=e^T\dot e=e^T(F+\Delta F+C_2Du)$$
$$=e^T(\Delta F-u_\Delta-u_k)$$
$$=\sum_{i=1}^p e_i(\Delta F_i-\hat\rho_i\tanh(\theta_ie_i)-k_ie_i)$$
$$\le\sum_{i=1}^p|e_i|(\rho_i-\hat\rho_i|\tanh(\theta_ie_i)|)-\sum_{i=1}^p k_ie_i^2. \tag{13}$$

Now, we consider the Lyapunov function as following:

$$V=V_1+\frac{1}{2}\sum_{i=1}^p(\rho_i-\hat\rho_i)^2. \tag{14}$$

By derivative of both sides of (14), we have

$$\dot V=\dot V_1+\sum_{i=1}^p(\rho_i-\hat\rho_i)\dot{\hat\rho}_i$$
$$=-\sum_{i=1}^p|e_i|\hat\rho_i(|\tanh(\theta_ie_i)|+1)-\sum_{i=1}^p k_ie_i^2$$
$$\le-\sum_{i=1}^p k_ie_i^2<0. \tag{15}$$

This completes the proof.

Now, we use

$$z_1=C_1(x), \tag{16}$$

$$z_2=C_2(y), \tag{17}$$

as the outputs of the master-slave systems (1) and (3) instead of Eq.(2) and (4), respectively. In this

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case, $C_1(x), C_2(x) \in R^p$ are the differentiable , function vectors.

Denotes

$$\bar{C}_1 = \frac{\partial C_1(x)}{\partial x}, \bar{C}_2 = \frac{\partial C_2(y)}{\partial y}, \quad (18)$$

$$\bar{F} = \bar{C}_2 g(y) - \bar{C}_1 f(x), \quad (19)$$

$$\Delta\bar{F} = [\Delta\bar{F}_1, \dots, \Delta\bar{F}_p]^T$$

$$= \bar{C}_2 \Delta g(y, \upsilon, t) - \bar{C}_1 \Delta f(x, \sigma, t). \quad (20)$$

The error of output-synchronization is given by

$$e = C_2(y) - C_1(x). \quad (21)$$

The derivative of both sides of (21) is presented

$$\dot{e} = \bar{F} + \Delta\bar{F} + \bar{C}_2 Du. \quad (22)$$

Assume that $\bar{C}_2 D$ is non singular, $|\Delta\bar{F}_i| \le \bar{\rho}_i$, $\bar{\rho}_i$ is positive constant. Then, we design following controller:

$$u = -(\bar{C}_2 D)^{-1}(\bar{F} + u_\Delta + u_k). \quad (23)$$

We gain the following conclusion:

Theorem 2. Consider the error system (22), With the control law u in Eq.(23) and adaptive law in Eq.(11), then, the error e in (22) converges to zero.

The proof is similar with theorem 1.

4. SIMULATIONS

In this section, we will provide an example to show the effectiveness the proposed method.

Consider following Lorenz-Stenflo chaotic system:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + cx_4, \\ \dot{x}_2 = rx_1 - x_2 - x_1x_3, \\ \dot{x}_3 = -bx_3 + x_1x_2, \\ \dot{x}_4 = x_1 + ax_4. \end{cases} \quad (24)$$

The system (24) exhibits chaos in Fig. 1, when the parameters $a = 2, b = 0.7, c = 1.5, d = 26$.

The Lorenz-Stenflo chaotic system with uncertainty as master system is given by:

$$\dot{x} = f(x) + \Delta f(x, t), \quad (25)$$

$$z_1 = C_1 x .$$

Where

$$f(x) = \begin{bmatrix} a(x_2 - x_1) + cx_4 \\ rx_1 - x_2 - x_1x_3 \\ -bx_3 + x_1x_2 \\ x_1 + ax_4 \end{bmatrix},$$

$$\Delta f(x, \sigma, t) = \begin{bmatrix} \Delta a(x_2 - x_1) + \Delta cx_4 \\ \Delta rx_1 \\ \Delta bx_3 \\ \Delta ax_4 \end{bmatrix} + \begin{bmatrix} 0.2\sin t \\ 0.3\cos 2t \\ 0.2\cos 3t \\ 0.3\sin 2t \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.5 & 0.2 & 0.2 & 0.1 \\ 0 & 0.5 & 0 & 0.2 \end{bmatrix},$$

$\sigma = [\Delta a, \Delta b, \Delta c, \Delta r]^T$, $\Delta a = 0.07\sin t$,

$\Delta b = 0.05\sin t$, $\Delta c = 0.08\cos t$, $\Delta r = 0.06\sin t$.

Consider following Lu chaotic system with uncertainties and input:

$$\dot{y} = g(y) + \Delta g(y, \alpha, t) + Du, \quad (26)$$

$$z_2 = C_2 y .$$

Where

$$g(y) = \begin{bmatrix} (25\alpha + 10)(y_2 - y_1) \\ (28 - 35\alpha)y_1 + (29\alpha - 1)y_2 - y_1y_2 \\ yy - (8 + \alpha)y_3 / 3 \end{bmatrix},$$

$$\Delta g(y, \upsilon, t) = \begin{bmatrix} \Delta\alpha(y_2 - y_1) \\ \Delta\alpha(y_1 + y_2) \\ \Delta\alpha y_3 \end{bmatrix} + \begin{bmatrix} 0.01\sin t \\ 0.02\sin 2t \\ 0.2\cos 4t \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0 & 0.4 & 0.2 \end{bmatrix}, D = \begin{bmatrix} 10 & 5 & 0 \\ 0 & 10 & 0 \end{bmatrix},$$

$\upsilon = \Delta\alpha = 0.01\sin 2t$. When parameter $\alpha = 0.8$,

and $\Delta g(y, \upsilon, t) = 0$, $u = 0$, the system (26) is chaos, which is shown in Fig.2. Now, by using control law (10) , adaptive law (11) and choosing $k = 5$, The simulation results of the proposed controller are shown in Fig. 3-5. From Fig. 3-5. it is seen that synchronous error converges to zero and controller is bounded.

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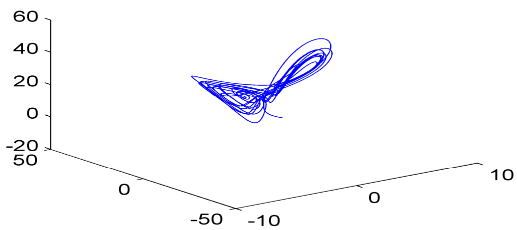

Fig.1: The Attractor Of Master System.

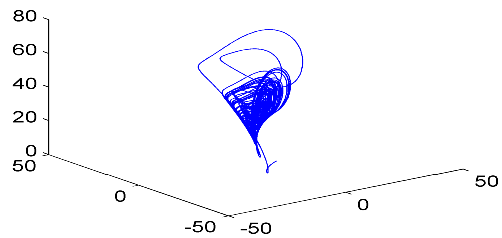

Fig.2: The Attractor Of Slave System.

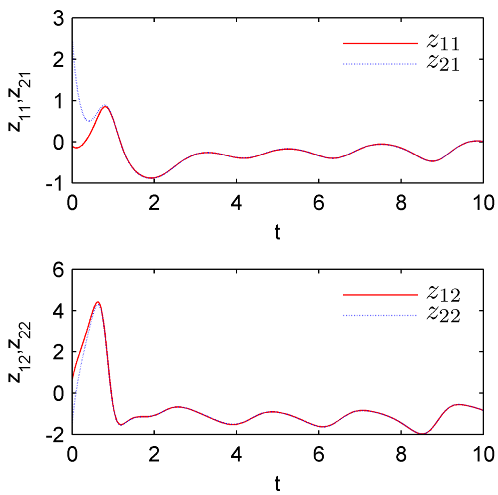

Fig.3: The Synchronization Outputs.

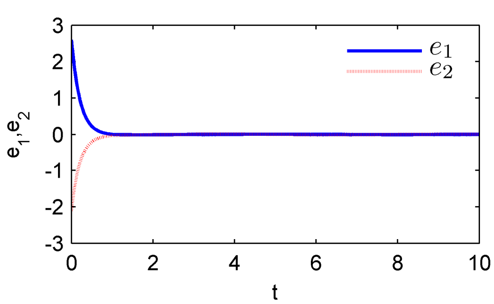

Fig.4: The Control Errors.

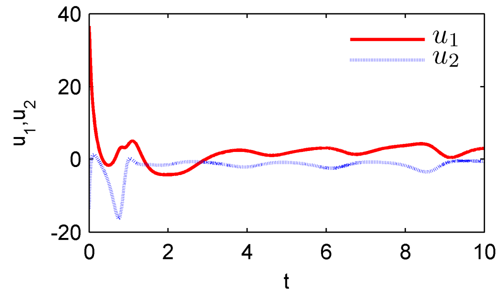

Fig.5: The Control Inputs.

In this paper, a new synchronization strategy is presented. An adaptive controller is given. The designed control law ensures that the error of the output-synchronization for different-order chaotic systems with uncertainties is stable. This method is in some sense an extension of the traditional synchronous ones. It may have some potential application in secure communication, etc.

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