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A NEW IMPROVED-MUSIC ALGORITHM FOR HIGH RESOLUTION DIRECTION OF ARRIVAL DETECTION

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ABSTRACT

This paper presents a new Improved-MUSIC (I-MUSIC) algorithm to solve the problem of precise direction of arrival detection. It is based on the actual MUSIC algorithm that has been modified in order to improve the resolution of the Direction of Arrival (DOA). The actual MUSIC algorithm suffers from power function that will reduce the resolution accuracy. For that reason, the I-MUSIC has been proposed by ignoring the covariance steering vector. The results show that this algorithm can give a high resolution even though the wireless system is operating under a very low Signal to Noise Ratio (SNR). Statistical comparisons with other algorithms (MUSIC, MUSIC-Like and MI-MUSIC) show that the I-MUSIC has the best performance.

Keywords: *Interference, Direction of Arrival, Signal to Noise Ratio, MUSIC Algorithm, Resolution.*

1. INTRODUCTION

The idea behind smart antenna system is to detect the DOA. This process can be done by emitting a narrow band signal from different sources and detect it back by sensor arrays using a specific algorithm. In the past three decades, a considerable amount of literatures have focused on the parametric subspace methods. The first serious discussions on the analyses of subspace-based DOA emerged during the 1986s when MUlti SIgnal Classification (MUSIC) algorithm has been proposed for the first time [1]. This algorithm uses the noise-subspace eigenvectors of the data correlation matrix in order to form a null spectrum and estimate the corresponding signal parameters. Mathematical modeling and analysis for the MUSIC power spectrum have been investigated with different signal to noise ratio (SNR), pilot coherent to largest local minimum values and the source parameter estimation as mentioned in [2].

In particular, many researchers have focused on developing a high resolution of MUSIC algorithm, which it has an attractive use in a critical mission of wireless services [3-5]. Higher resolution DOA estimation can be obtained by MUSIC algorithm, which can give a better performance compared to Maximum likelihood method and estimation of signal parameters via the rotational invariance techniques (ESPRIT) algorithm [6].

Another method such as a high order cumulative method requires signal statistical properties and it needs larger snapshots in order to have a precise DOA detection. In addition it suffers from heavy computation loads which make the mobile system complicated and unfeasible design. Alternatively, it has been proved that MUSIC-Like algorithm can exploit the cyclostationarity in order to increase the resolution power and noise robustness. However, both cyclic and conjugate cyclic correlation matrices are involved in the algorithm which increases the complexity and only applicable with limited conditions [7]. The Conjugate Augmented MUSIC (CAM) algorithm mentioned in [8] is a second-order statistical approach of the received signals to get the conjugate steering matrix, together with steering matrix, is used to find the fourth-order cumulates. CAM is better than MUSIC-Like algorithm in terms of number of directions, estimation capacity, angle resolution, required snapshots and immunity to noise. In [9] multi-invariance MUSIC (MI-MUSIC) algorithm has been introduced to prove that it has a better angle and delay estimation performance than ESPRIT and MUSIC algorithm.

This paper is organized as follows. First, we identify the steering vector obtained using the original MUSIC algorithm described in Section 2. Then, we derive the Improved MUSIC (I-MUSIC) algorithm, which distinctively reduced the complexity of DOA estimation as introduced in Section 3. Next, we compare the root mean squared

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error (RMSE) and costs of computational complexity against the conventional algorithms in Section 4. The proposed I-MUSIC algorithm can have better performance in terms of resolution detection and less SNR required compared to the MUSIC, MI-MUSIC and MUSIC-Like algorithms. Finally conclusions are drawn in Section 5. The proposed I-MUSIC algorithm can have better results than MUSIC, MI-MUSIC and MUSIC-Like algorithms in terms of resolution, performance and less SNR required. Numerical results for different array antenna manifolds and a variety of data lengths are also presented in the simulations.

Notation: $(.)^*$ denote the complex conjugation, $(.)^T$ denote the transpose function, $(.)^H$ denote the conjugate transpose function and E is the statistical expectation process. σ^2 is the noise standard deviation, $\mathbf{I_Q}$ is a $Q \times Q$ identify matrix, and $\| \ \|_\mathbf{F}$ is the Frobenius Norm.

2. STEERING VECTOR

In order to derive the steering vector, let $\mathbf{x}(t)$ to be the vector process of the complex signals at the output and M is the number of array for the narrow band identical sensors. A specific number of rays P have been transmitted to M, using half-wave length spacing denotes as a P-dimentional source vector. The received base-band signal can be expressed as:

$$\mathbf{x}(t) = \sum_{P=1}^{P} s_p \mathbf{a}(\theta_\mathbf{P}) + \mathbf{n}(t) \\ = \mathbf{A}(\Theta)\mathbf{S}(t) + \mathbf{n}(t) \tag{1}$$

Where $\mathbf{S}(t) = [s_1(t),...,s_p(t)]^T$ is the useful data inside the Pth array, $\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1),...,\mathbf{a}(\theta_P)]$ is the array direction vector for DOA θ_P of the Pth ray. It is assumed that all sources emit a common pulse shape.

In order to represent a number of data for specific time duration, we considered an only one sample deploying b times symbol rate during C periods of time. As a result the samples matrix is $MC \times b$:

$$MC \times b = \begin{bmatrix} x(0) & x(1) & . \ . & x(\mathbf{C}-1) \\ x\left(\frac{1}{b}\right) & x\left(1+\frac{1}{b}\right) & . \ . & x(\mathbf{C}-1+\frac{1}{b}) \\ . & . & . \ . & . \\ . & . & & . \\ x\left(1-\frac{1}{b}\right) & x\left(2+\frac{1}{b}\right) & . \ . & x(\mathbf{C}-\frac{1}{b}) \end{bmatrix} = \mathbf{X} \tag{2}$$

Next we applied Fourier transform to the sampled output. Then we find the covariance matrix $\mathbf{R}_x$ which emitted by $\bar{\mathbf{X}}\bar{\mathbf{X}}^H / \mathbf{C}$, as follow:

$$\hat{\mathbf{R}}_x = E\left[\mathbf{x}(t)\mathbf{x}(t)^H\right] = \mathbf{E}_s\mathbf{D}_s\mathbf{E}_s^H + \mathbf{E}_n\mathbf{D}_n\mathbf{E}_n^H \\ = \mathbf{S}\mathbf{R}_{xx}\mathbf{S}^* + \sigma^2\mathbf{I_Q} \\ = \begin{bmatrix} \mathbf{R}_{x0} & . \ . \ . & \mathbf{R}_{xP} \\ . & . & . \\ . & . & . \\ . & . & . \\ \mathbf{R}_{xP} & . \ . \ . & \mathbf{R}_{x0} \end{bmatrix} \tag{3}$$

Where $\mathbf{E}_n$ stands for subspace noise component, $\mathbf{D}_s$ is a $P \times P$ diagonal matrix including the largest P eigenvalues, and $\mathbf{D}_n$ is a diagonal matrix for a smallest eigenvalues. Therefore, $\mathbf{E}_s$ will response to the largest P eigenvalues of $\hat{\mathbf{R}}_x$. $\mathbf{E}_n$ represents matrix including the rest of eigenvectors. Then after, we identify the steering vector which corresponding values in angles, as follow:

$$\mathbf{A} = e^{\frac{J2\pi d}{\lambda}\cos(\theta_P)\times\frac{\pi}{180}\times\{0...M-1)} \tag{4}$$

Where λ is a wavelength and d is the space between two elements, we consider d= 0.5 λ .

3. IMPROVED MUSIC ALGORITHM FOR HIGH-RESOLUTION

The complexity of Eigenvalue decomposition is the main difficulty in order to process MUSIC algorithm in real time deployment. This is mainly due to its computationally-intensive requirements. Thus, it is very important to reproducing the MUSIC spectrum in order to improve the overall resolution. According to Eq. 3,

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firstly we construct the MUSIC spectrum function which computed for 180^0:

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\mathbf{a}(\theta)} \quad (5)$$

Where $\mathbf{a}(\theta)$ is:

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 & e^{-\frac{-j2\pi d}{\lambda}\cos\left(\frac{a\pi}{180}\right)} & . & . & . \\ e^{-\frac{-j2\pi d}{\lambda}(M-1)\cos\left(\frac{a\pi}{180}\right)} & & & & \end{bmatrix} \quad (6)$$

Based on the Eq. 5, many literatures have tried to improve it. However, most of the improvements on MUSIC algorithm require an exhaustive searching, which is inefficient due to high computational cost. In this section, we propose a new I-MUSIC algorithm, which is qualified for high-resolution detection using one-dimension of searching.

I-MUSIC algorithm is based on the fact that covariance of steering vector is actually not necessary because it can increase the power function. However, in the same time, it can reduce the accuracy to reach the mean. Instead, our objective is to increase the resolution. Therefore, noise subspace correlation matrix will be correlated with vector mode in order to represent the power from the subspace. Finally, the new proposed I-Music spectrum is:

$$PMU(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\left|1 \quad 0 \quad . \quad . \quad (M-1)\right|} \quad (7)$$

This expression is rather similar to the one given in Eq. 6. However our method is dedicated to have extra high resolution that what proposed in [1]. Added to that, we can use it under low condition of SNR with a quite impressive resolution. It should be emphasized that our proposed method has better estimation performance than MUSIC-Like and MI-MUSIC which proposed in [10]. The idea of the next formulas is adopted to estimate the DOA without noise is presented. The signal subspace can be denoted as:

$$\mathbf{E}_s = \mathbf{Y}\begin{bmatrix} \mathbf{A}_\theta\mathbf{D}_1 \\ . \\ . \\ \mathbf{A}_\theta\mathbf{D}_b \end{bmatrix} = \mathbf{Y}\begin{bmatrix} \mathbf{A}_\theta \\ . \\ . \\ \mathbf{A}_\theta\Phi^{b-1} \end{bmatrix} = \Lambda\mathbf{Y} \quad (8)$$

$$= [\mathbf{a}(\theta_1),. \quad . \quad .,\mathbf{a}(\theta_p)]\ \mathbf{Y}$$

Where $\mathbf{Y}$ is a $P \times P$ matrix, Φ is the rotational matrix and equal to diagonal of: $[\exp(-j2\pi / b). \quad . \quad .\exp(-j2\pi / b_p)]$.

According to Eq. 8, diagonal can be calculated as: $\Lambda = \mathbf{E}_s\mathbf{Y}^{-1}$, while signal subspace fitting can be found from the argument minimum as:

$$\hat{\Pi}, \Lambda = \arg_{\mathbf{Y}}^{\min}\left\|\Lambda - \hat{\mathbf{E}}_s\Pi^{-1}\right\|_F^2 \quad (9)$$

Where $\hat{\mathbf{E}}_s$ is estimator of $\mathbf{E}_s$, that can be attained by searching for the deepest $\mathbf{V}$ minimum as:

$$P = a(\theta)^H\mathbf{Q}\left|1 \quad 0 \quad . \quad . \quad .(M-1)\right| \quad (10)$$

Where: $\mathbf{Q} = \mathbf{I}_M^H\Pi_{\hat{E}_s}^{90^0}\mathbf{I}_M$. Eq. 10 is representing the problem of quadratic optimization. We consider the constraint of $e_1^Y a(\theta) = 1$, where $e_1 = [1 \quad , \quad 0 \quad ,. \quad . \quad ,0]^Y \in \mathbf{R}^{M\times 1}$ has been added to estimate the trivial solution $a(\theta) = \mathbf{0}_M$. The optimization problem can be reconstructed with the linear constraint minimum variance solution. We can get the actual DOA by scanning the angle from 0 to 180^0, where V's are the largest peaks correspond to the DOA detection. In order to achieve I-MUSIC resolution, we summarized the operational process in three steps as follows:

Step1: Calculate the covariance matrix for the received signal, in order to measure how many variables are changing together.

Step2: Calculate the eigen-decomposition in order to associate the covariance in a square matrix.

Step 3: Search for the largest peaks $P's$ of the $b(\theta)^{-1}$ elements at the required DOA signals. In contrast, our algorithm can have same computational load to MUSIC and lower complexity than MUSIC- Like and MI-MUSIC.

4. SIMULATION RESULTS

A random Binary phase shift keying (BPSK) signal is generated and accompanied by uncorrelated random noise of the arrival signals. Then, a data matrix as in Eq. 3 will be generated and auto correlated to get the covariance matrix. In the following simulation we assume there are four coherent rays to arrive at array antenna. Their DOAs are 75^0, 90^0, 105^0 and 120^0. The estimated sources number based on Akaike's information criterion by calculating the root minimum square error $RMSE = \sqrt{\frac{1}{200}\sum_{n=1}^{200}(x-\bar{x})^2}$, where x is the obtained correct angle of arrival for P^{th} array of

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20^0 independent experiment and $\bar{x}$ is the perfect angle of arrival.

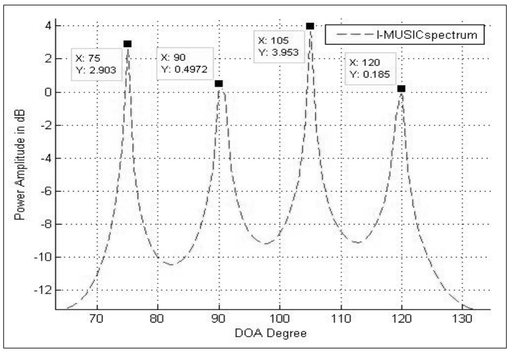

Figure 1. The angle estimation performance with I-MUSIC at SNR=15dB.

Figure 1 shows the angle estimation for I-MUSIC algorithm (the number of sensors used are M=8, times symbol rate b=10, number of periods, C=100 and SNR=15dB). The observed spectrum peaks can detect the angles accurately, which describe that the algorithm is working well. Figure 2 shows the estimation angle performance of I-MUSIC algorithm (using M=8, b=10 and C=50) compared with MUSIC, MI-MUSIC and MUSIC-Like methods. It is indicated that among the four algorithms I-MUSIC gives the best detection resolution in the environment, including noise and fading. The accuracy level of the angles which is near to each other is evaluated. It is observed that the highest accuracy is for the new proposed I-MUSIC algorithm and the lowest accuracy is for the conventional MUSIC algorithm. Where, the lowest accuracy for normal MUSIC is due to the natural processes of the mean calculation for this algorithm.

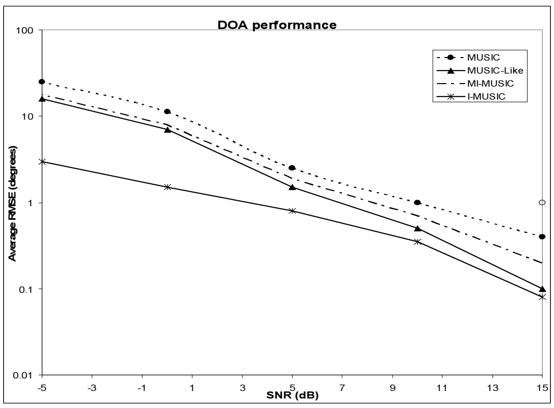

Figure 2. The Angle resolution accuracy comparison with C=50.

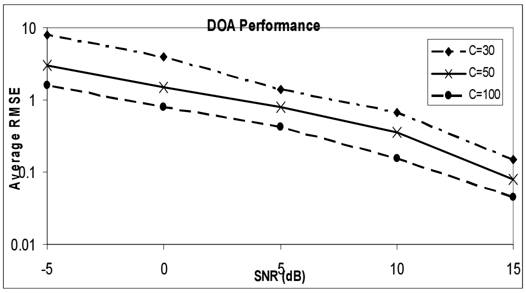

Figure 3. Angle estimation with different C.

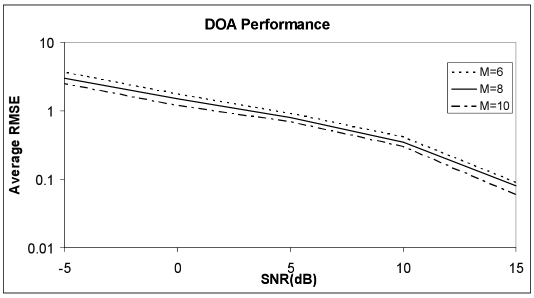

Figure 4. Angle estimation with C=50 and different M.

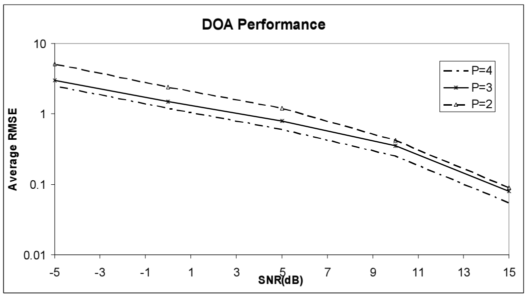

Figure 5. Angle estimation with with C=50 and different numbers of P.

MUSIC has the worst estimation angle among the four algorithms. Figure 3 depicts the algorithmic performance comparisons where MI-MUSIC has been adopted. The simulation is shown for different C (the same M = 8 and b = 10 as in Figure 2). It can be seen that the MI-MUSIC follows the same principle as MUSIC. However, the enhancement is only done on the multi signals time invariance. It indicates that the estimation angles are better when C is increasing. Figure 4 illustrates the estimation angle for I-MUSIC algorithm using different number of arrays. It shows that the angle estimation is gradually improved with the increased number of antennas. At a specific number of antennas, further increment of M will be useless. The increased number of M could be unfeasible comparing to the

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estimation improvement that we have achieved. However, multiple antennas improve the estimation angle because the used of diversity gain. Figure 5 displays the algorithm performance of I-MUSIC under different P (when M = 8, b = 10, and C = 100). It shows that the estimation angles are gradually decreased when the numbers of sources are increased. In the same time, it will increase the number of snapshots that will habitually boost up the resolution of proposed algorithm.

5. CONCLUSION

A new I-MUSIC algorithm has been proposed for the detection of high resolution DOA. Low computational cost is the advantage of this algorithm compared to the other subspace-base algorithms. This algorithm has been derived from the conventional MUSIC algorithm and it provides a very good performance especially in estimating the angles of DOA compared to MUSIC, MI-MUSIC and MUSIC-Like algorithms. In the future, I-MUSIC can systematically develop to work with any other form of arrays in order to represent sustainable high resolution detection.

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