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OSCILLATION EQUATIONS OF RECTANGULAR PLATES IN LINEAR APPROXIMATION

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ABSTRACT

This work discusses general oscillation equations of viscoelastic isotropic medium in linear approximation with and without consideration for initial shifts and stresses, as well as approximate oscillation equations with consideration for ambient environment and at small deformations. Under the impact of forces applied to continuous deformed body, the positional relationship of its particles varies, that is, the deformed body varies its shape and volume. This can be exemplified by compressed or tensioned rod. Mathematical description of deformation of solid body is given in this or that coordinate system. For instance, in Cartesian coordinates $x_1=x;\ x_2=y;\ x_3=z$ position of each point is defined by radius vector $\vec{r}$ and the components $(x_1,\ x_2,\ x_3)$. After deformation, position of the point will be defined by another coordinates: $(x_1',\ x_2',\ x_3')$.

Keywords: *Longitudinal Oscillations, Transversal Oscillations, Viscoelastic Plate, Deformed Body, Small Deformation.*

1. INTRODUCTION

This work presents fundamental results by Kazakhstan and foreign scientists. The authors mention only certain major works based on the most popular mathematical models. Fundamental concepts and approaches to development of mathematical models, theoretical and experimental studies in the field of dynamic interaction of plate and base are related with such scientists as Akhenbakh, Vlasov, Grigolyuk, Ilyushin, Lentev, Petrashen, Rakhmatullin, Timoshenko, Filippov, Tyurekhodzhaev and others [1, 2].

Numerous urgent scientific and engineering problems are related with investigations into oscillation processes and wave propagation in continuous mediums. The obtained results are applied upon consideration of nonstationary oscillation and wave processes. However, there are some questions related with response of medium to external impacts, methods of motion initiation, kinematic properties of waves, body geometry, solution of these problems is important for practical use and achieved by peculiar methods typical for this field.

While solving the application problems of oscillation of rectangular plates, it is obvious that

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there exists wide range of relevant problems related with various boundary conditions: approximate oscillation equations at plate edges and initial conditions. In the oscillation theory an important issue is determination of eigenfrequency, solution of problems of forced oscillation of plate and analysis of propagation of harmonic waves.

This work presents the required data on theory of elasticity and viscoelasticity, the main boundary problems of viscoelastic medium dynamics are formulated. Two-component viscoelastic mediums are considered at small deformations.

2. METHODS

Exact equations of longitudinal and transversal oscillations of viscoelastic plates with and without consideration for initial shifts and stresses, as well as approximate oscillation equations with consideration for ambient environment and at small deformations have been derived using mathematical approach [3].

Under the impact of forces applied to continuous deformed body, the positional relationship of its particles varies, that is, the deformed body varies its shape and volume. This can be exemplified by compressed or tensioned rod.

Mathematical description of deformation of solid body is given in this or that coordinate system. For instance, in Cartesian coordinates $x_1=x;\ x_2=y;\ x_3=z$, position of each point is defined by radius vector $\vec{r}$ and the components $(x_1,\ x_2,\ x_3)$. After deformation, position of the point will be defined by another coordinates: $(x_1',\ x_2',\ x_3')$.

Vector $\vec{u}=\vec{r}'-\vec{r}$ defines the vector of point displacement, and the coordinates $(x_1',\ x_2',\ x_3')$ are the functions of initial coordinates $(x_1,\ x_2,\ x_3)$. Hence, the vector of displacement is also the function of coordinates $(x_1,\ x_2,\ x_3)$.

3. RESULTS AND DISCUSSION

Let us consider any two close points with the radius vector $(dx_1, d\,x_2, d\,x_3)$ between them positioned at the distance of $dl=\sqrt{dx_1^2+dx_2^2+dx_3^2}$. After deformation, the distance between the points is $dl'=\sqrt{(dx_1')^2+(dx_2')^2(dx_3')^2}$, where $dx_j'=dx_j+dU_j$, U_j are the coordinates of displacement vector.

Writing in brief form $dl^2=(dx_j)^2;\ dl'^2=(dx_j')^2$ and substituting $du_j=\frac{\partial u_j}{\partial x_k}dx_k$, we obtain $(dl')^2=dl^2+2\frac{\partial u_j}{\partial x_k}dx_k+\frac{\partial u_j}{\partial x_k}\frac{\partial u_j}{\partial x_i}dx_k dx_i$.

After elementary transformations, we have:

$$(dl')^2=(dl)^2+2\gamma_{jk}\,dx_j dx_k;\ \gamma_{jj}=\varepsilon_{jj};\ \gamma_{jk}=\frac{1}{2}\varepsilon_{jk};$$

where

$$\gamma_{jk}=\frac{1}{2}\left(\frac{\partial u_j}{\partial x_k}+\frac{\partial u_k}{\partial x_j}+\frac{\partial u_n}{\partial x_k}+\frac{\partial u_n}{\partial x_j}\right). \quad (1)$$

Since we will consider only small deformations, then in Eq. (1) we neglect products of derivatives of displacements along coordinates and assume approximately:

$$\varepsilon_{jj}=\frac{\partial u_j}{\partial x_j};\quad \varepsilon_{jk}=\frac{\partial u_j}{\partial x_j}+\frac{\partial u_k}{\partial x_j};\quad (j\neq k)\ . \quad (2)$$

Small deformations defined by Eq. (2) form deformation tensor:

$$D=\begin{pmatrix} \varepsilon_{11}; & \frac{1}{2}\varepsilon_{12}; & \frac{1}{2}\varepsilon_{13} \\ \frac{1}{2}\varepsilon_{21}; & \varepsilon_{22}; & \frac{1}{2}\varepsilon_{23} \\ \frac{1}{2}\varepsilon_{31}; & \frac{1}{2}\varepsilon_{32}; & \varepsilon_{33} \end{pmatrix};\quad \varepsilon_{jk}=\varepsilon_{kj} \quad (3)$$

The deformation tensor, Eq. (3), has three independent invariants:

$$\tau_1=\varepsilon_{11}+\varepsilon_{22}+\varepsilon_{33}; \quad (4)$$

$$\tau_3=|D|$$

$$\tau_2=\varepsilon_{11}\varepsilon_{22}+\varepsilon_{11}\varepsilon_{33}+\varepsilon_{22}\varepsilon_{33}-\frac{1}{4}\left(\varepsilon_{12}^2+\varepsilon_{13}^2+\varepsilon_{23}^2\right)$$

where the invariant τ_1 is referred to as bulk extension.

Let us introduce three main elongations, $\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3$, then the invariants τ_1, τ_2, τ_3 are:

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$$\tau_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3;$$
$$\tau_2 = \varepsilon_1\varepsilon_2 + \varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_3;$$
$$\tau_3 = \varepsilon_1\varepsilon_2\varepsilon_3;$$

herewith, the elongations ε_1, ε_2, ε_3 are the roots of cubic equation:

$$\varepsilon^3 - \tau_1\varepsilon^2 + \tau_2\varepsilon - \tau_3 = 0.$$

Using the main elongations, the average elongation is defined:

$$\varepsilon_0 = \frac{1}{3}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = \frac{1}{3}\tau_1$$

and respective spherical tensor is as follows:

$$D_0 = \frac{1}{3}\tau_1 E = \varepsilon_0 E = \begin{pmatrix} \varepsilon_0,\ 0,\ 0 \\ 0, \varepsilon_0,\ 0 \\ 0,\ 0, \varepsilon_0 \end{pmatrix} \quad (5)$$

The difference between the tensors D and D_0 is referred to as deviatoric tensor of deformation equaling to [4]:

$$D' = \begin{pmatrix} \varepsilon_{11} - \varepsilon_0; & \frac{1}{2}\varepsilon_{12}; & \frac{1}{2}\varepsilon_{13} \\ \frac{1}{2}\varepsilon_{21}; & \varepsilon_{22} - \varepsilon_0; & \frac{1}{2}\varepsilon_{23} \\ \frac{1}{2}\varepsilon_{31}; & \frac{1}{2}\varepsilon_{32}; & \varepsilon_{33} - \varepsilon_0 \end{pmatrix} \quad (6)$$

where main elongations can also be introduced:

$$\varepsilon_1' = \varepsilon_1 - \varepsilon_0;$$
$$\varepsilon_2' = \varepsilon_2 - \varepsilon_0;$$
$$\varepsilon_3' = \varepsilon_3 - \varepsilon_0; \quad ,$$
$$\varepsilon_1' + \varepsilon_2' + \varepsilon_3' = 0$$

that is, the deviator D' determines deformation without bulk extension or deformation of type variation without variation of volume.

Intensity of shear deformation is very important in the deformation theory, it is determined as follows:

$$\Psi_0^2 = \frac{2}{\sqrt{3}}\left[\frac{2}{3}\left(\varepsilon_{11}^2 + \varepsilon_{22}^2 + \varepsilon_{33}^2 - \varepsilon_{11}\varepsilon_{22} - \varepsilon_{11}\varepsilon_{33} - \varepsilon_{22}\varepsilon_{33}\right) + \frac{1}{2}\left(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2\right)\right] \quad (7)$$

In particular, deformations, Eq. (2), in cylindrical coordinates are as follows:

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}; \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}; \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z};$$
$$\varepsilon_{\theta z} = \frac{\partial u_\theta}{\partial z} + \frac{\partial u_z}{\partial \theta}; \quad \varepsilon_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}; \quad (8)$$
$$\varepsilon_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}.$$

Equations (7) and (8) will be used for solution of the problems.

In order to describe stressed state of continuous body, let us consider an arbitrary point M inside the body and possible sites $d\sigma$ in this point.

This site is affected from the side of medium by the force denoted as $d\vec{P}$.

Let us assume:

$$d\vec{P} = \vec{P}_n d\sigma, \quad (9)$$

where $\vec{P}_n$ is the finite vector, $\vec{n}$ is the normal to the site $d\sigma$.

Expanding the force $\vec{P}_n$ into constituents by the normal $\vec{n}$ and by the tangential $\vec{\tau}$ to the site $d\sigma$, we obtain:

$$\vec{P}_n = \sigma_{nn}\vec{n} + \sigma_{n\tau}\vec{\tau} \quad (10)$$

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where σ_{nn} is the normal stress, $\sigma_{n\tau}$ is the tangential stress.

Since infinite number of sites $d\sigma$ can be plotted across the point M, then, there exists infinite number of forces $\vec{P}_n$ corresponding to these sites. However, among these forces only three are linearly independent, all other forces can be expressed on their basis. We will consider the forces acting on sites perpendicular to the selected orthogonal coordinates as independent. These forces as three independent vectors form the stress tensor [5]:

$$T = \begin{pmatrix} \sigma_{xx}, \sigma_{xy}, \sigma_{xz} \\ \sigma_{yx}, \sigma_{yy}, \sigma_{yz} \\ \sigma_{zx}, \sigma_{zy}, \sigma_{zz} \end{pmatrix}, \quad (11)$$

herewith, this tensor is symmetrical, since it is assumed that the pairing law of tangential deformations $\sigma_{ij} = \sigma_{ji}$ is valid.

For the deformation tensor (11), it is possible to introduce the notion of main stresses $\sigma_1, \sigma_2, \sigma_3$ and invariants equaling to:

$$S_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}; \qquad S_3 = |T|;$$
$$S_2 = \sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz} - \sigma^2{}_{xy} - \sigma^2{}_{xz} - \sigma^2{}_{yz},$$

and these main stresses are the roots of cubic equation:

$$\sigma^3 - S_1\sigma^2 + S_2\sigma - S_3 = 0$$

By introducing the average stress $\sigma_0 = \frac{1}{3}S_1$, the tensor (11) can be subdivided into two constitutes:

$$T = T_0 + T'$$

where T_0 is the spherical tensor or the tensor of hydrostatic stress, and T' is the deviatoric tensor equaling to:

$$T_0 = \begin{pmatrix} \partial_0, 0, 0 \\ 0, \partial_0, 0 \\ 0, 0, \partial_0 \end{pmatrix};$$
$$T' = \begin{pmatrix} \partial_{xx} - \partial_0; \partial_{xy}; & \partial_{xz} \\ \partial_{yx}; & \partial_{yy} - \partial_0; \partial_{yz} \\ \partial_{zx}; & \partial_{zy}; & \partial_{zz} - \partial_0 \end{pmatrix} \quad (12)$$

The notion of intensity of tangential stresses is very important for analysis of stressed state in a body:

$$\tau_0^2 = \sqrt{\frac{2}{3}}\left[\frac{1}{3}\left(\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 - \sigma_{xx}\sigma_{yy} - \sigma_{xx}\sigma_{zz} - \sigma_{yy}\sigma_{zz}\right) + \sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2\right]$$

The introduced in this work notions and values completely characterize stressed-strained state of continuous body in any point in the case of small deformations [6].

Knowing the properties of the introduced variables characterizing stressed-strained state, let us formulate the laws relating these properties of viscoelastic body at small deformations.

Let us mention nonlinear law of $\sigma_{ij} \sim \varepsilon_{ij}$ for elastic isotropic body. Brief conclusion will be presented below.

At first the linear law of $\sigma_{ij} \sim \varepsilon_{ij}$ is given which can be written as follows:

$$\sigma_0 = 3K\varepsilon_0; \qquad T' = 2GD', \quad (13)$$

where K is the bulk compression modulus, G is the shearing modulus, which are related with the Láme constants as follows:

$$K = \lambda + \frac{2}{3}\mu; \qquad G = \mu. \quad (14)$$

Let us write the nonlinear law of $\sigma_{ij} \sim \varepsilon_{ij}$ for small deformations such that in the limit for infinitely small deformations, it is transformed into the Hooke's law (13).

Let us consider specific work of deformation:

$$A = \int_0^D \left(\sigma_{xx} d\varepsilon_{xx} + \sigma_{yy} d\varepsilon_{yy} + \sigma_{zz} d\varepsilon_{zz} + \sigma_{xy} d\varepsilon_{xy} + \sigma_{xz} d\varepsilon_{xz} + \sigma_{yz} d\varepsilon_{xz}\right) \quad (15)$$

In this case, integration is carried out from the state when all components of deformation are zero and to the state when they are presented by the tensor D.

If to assume $\sigma_{xx} = \sigma_{x'x} + \sigma_0, \ldots$ and consider that:

$$\sigma_{x'x} + \sigma_{y'y} + \sigma_{z'z} = 0;$$
$$d\varepsilon_{x'x} + d\varepsilon_{y'y} + d\varepsilon_{z'z} = 0$$

then:

$$A(x,y,z) = A_0(x,y,z) + A'(x,y,z),$$

where

$$A_0 = 3\int_0^D \sigma_0 d\varepsilon_0 \quad (16)$$

is the work of volume variation and

$$A'(x,y,z) = \int_0^D (\sigma_{xx'} d\varepsilon_{xx'} + \sigma_{yy'} d\varepsilon_{yy'} + \sigma_{zz'} d\varepsilon_{zz'} + \sigma_{xy} d\varepsilon_{xy} + \sigma_{xz} d\varepsilon_{xz} + \sigma_{yz} d\varepsilon_{xz}) \quad (17)$$

is the work of shape variation.

In order to derive nonlinear function $\sigma_{ij} \sim \varepsilon_{ij}$, it is possible to satisfy the conditions [7]:

1. Specific work of deformation A should be simultaneously the function of components of the deformation tensor D.
2. Material of the body should be homogeneous and isotropic.
3. The spherical tensor of stress T_0, as in the Hooke's law, should depend only on the spherical deformation tensor D_0, and the deviator of stress tensor T' - on the deviator of deformation tensor D'.
4. For infinitely small deformation, the established law should be of the same form as that of the Hooke's law (13).

Since the formulated conditions should be satisfied, for specific work A we have [8]:

$$A = A_0(\varepsilon_0) + A'(\psi_0^2, \tau_3'), \quad (18)$$

where τ_3' is the third invariant of deviator of deformation tensor.

As demonstrated in [9], the variable A' should not depend on the invariant τ_3', and then the nonlinear law of $\sigma_{ij} \sim \varepsilon_{ij}$ will be as follows:

$$\sigma_0 = 3K\varepsilon_0 x_0(\varepsilon_0);$$
$$T' = 2G\gamma(\psi_0^2)D'$$

(19)

where x_0 is the equation function, and γ is the shear function; using the constituents of specific work of deformation, they are expressed as follows:

$$x_0(\varepsilon_0) = \frac{1}{9K\varepsilon_0}\frac{dA_0}{d\varepsilon_0};$$
$$\gamma(\psi_0^2) = \frac{4}{3G}\frac{dA'}{d\psi_0^2}. \quad (20)$$

From Eq. (19) for separate components of the stress tensor, we obtain:

$$\sigma_{jj} = 3K\varepsilon_0 x_0(\varepsilon_0) + 2G(\varepsilon_{jj} - \varepsilon_0)\gamma(\psi_0^2);$$
$$\sigma_{ij} = G\varepsilon_{ij}\gamma(\psi_0^2);$$
$$(i \neq j;\ i,j = x,y,z). \quad (21)$$

Further, it would be more convenient to use the functions of elongation and shear in the following form:

$$x_0(\varepsilon_0) = 1 + f_0(\varepsilon_0); \qquad \gamma(\psi_0^2) = 1 + f_1(\psi_0^2);$$
$$f_j(0) = 0, \qquad (22)$$

and then Eq. (21) is rewritten as follows:

$$\sigma_{jj} = 3K\varepsilon_0[1 + f_0(\varepsilon_0)] + 2G(\varepsilon_{jj} - \varepsilon_0)[1 + f_1(\psi_0^2)];$$
$$\sigma_{ij} = G\varepsilon_{ij}[1 + f_1(\psi_0^2)]; \quad (i \neq j). \quad (23)$$

Since Eq. (23) should be the same both upon loading and unloading, the function $f_0(\varepsilon_0)$ should be even with regard to ε_0, and $f_1(\psi_0^2)$ should depend only on ψ_0^2. In particular, upon expansion of these functions into exponential series, we have as follows:

$$f_0(\varepsilon_0) = \sum_{n=0}^{\infty} \alpha_n \varepsilon_0^{2(n-1)};$$
$$f_1(\psi_0^2) = \sum_{n=0}^{\infty} \gamma_n \psi_0^{2(n+1)}. \qquad (24)$$

Linear theory of viscoelasticity is based on the memory effect, that is, on linear integral dependence of stresses on deformations. Then, Eq. (13) for linear elastic body can be written for linear viscoelastic body as follows:

$$\sigma_0 = 3KR_0(\varepsilon_0); \qquad T' = 2GR(D'), \quad (25)$$

where R_0 and R are the linear integral operators of Volterra type:

$$R_0(\zeta) = \zeta(t) - \int_0^t F_{10}(t-\xi)\zeta(\xi)d\xi;$$
$$R(\zeta) = \zeta(t) - \int_0^t F_{20}(t-\xi)\zeta(\xi)d\xi; \qquad (26)$$

$F_{j0}(t)$ are the kernels of these operators.

As in the case of Eq. (13), the specific work of deformation A can be represented as follows:

$$A = A_0 + A'. \qquad (27)$$

For generalization of Eq. (21) for the case of viscoelastic body, let us assume that:

1. The specific work of deformation A is the single valued function of overall deformation history to the current time t;
2. The material of viscoelastic body is homogeneous and isotropic;
3. The deviator of stress tensor T' depends only on the variation history of the deviator D', and the average stress σ_0 depends only on the variation history of average deformation ε_0.

For infinitely small deformations, the nonlinear law of $\sigma_{ij} \sim \varepsilon_{ij}$ in the limit should be transferred to the law of linearity of the viscoelastic theory.

On the basis of the mentioned conditions, we have for the specific work of deformation A:

$$A(x,y,z) = \rho_0(I_1) + \rho_1(\psi_0^2), \qquad (28)$$

where ρ_0 and ρ_1 are the nonlinear functionals.

Then, the generalized nonlinear law of $\sigma_{ij} \sim \varepsilon_{ij}$ for viscoelastic body will be as follows:

$$\sigma_0 = 3KR_0[r_0(\varepsilon_0)\varepsilon_0];$$
$$T' = 2GR[r_0(\psi_0^2)D'] \qquad (29)$$

where

$$R_0 = [r_0(\varepsilon_0)\varepsilon_0] = \frac{1}{9K}\frac{d\rho_0}{d\varepsilon_0};$$
$$R[r(\psi_0^2)D'] = \frac{4}{3G}\frac{d\rho_1}{d\psi_0^2}D'.$$

On the basis of isotropy, the operators r_0 and r can be presented in the series:

$$r_0(\varepsilon_0) = 1 + \sum_{n=1}^{\infty} K_n(\varepsilon_0^n)$$
$$r(\psi_0^2) = 1 + \sum_{n=1}^{\infty} G_n(\psi_0^{2n}), \qquad (30)$$

where

$$K_n(\varepsilon_0^n)=\alpha_n\left[\varepsilon_0^{2n}(t)-\int_0^t\dots\int_0^t F_{1n}(t-\xi_1,\dots,t-\xi_n)\times\varepsilon_0^2(\xi_1),\dots,\varepsilon_0^2(\xi_n)d\xi_1,\dots,d\xi_n\right];$$

$$G_n(\psi_0^{2n})=\gamma_n\left[\psi_0^{2n}(t)-\int_0^t\dots\int_0^t F_{2n}(t-\xi_1,\dots,t-\xi_n)\times\psi_0^2(\xi_1),\dots,\psi_0^2(\xi_n)d\xi_1,\dots,d\xi_n\right];$$

(31)

F_{jn} are the n-dimensional kernels of the integral operators K_n and G_n.

For stress components, the law for viscoelastic body can be rewritten as follows:

$$\sigma_{jj}=3KR_0[r_0(\varepsilon_0)\varepsilon_0]+2GR[r(\psi_0^2)(\varepsilon_{jj}-\varepsilon_0)];$$

$$\sigma_{ij}=GR[r(\psi_0^2)\varepsilon_{ij}];$$

$$i\neq j;\quad i,j=x,y,z \qquad (32)$$

The latter expressions are nonlinear interrelations between σ_{ij} and ε_{ij} for small deformations.

In arbitrary curvilinear orthogonal coordinates (α,β,γ), displacement of elastic or viscoelastic medium upon small deformations is described by stress equations:

$$\frac{\partial}{\partial\alpha}(h_2h_3\sigma_{\alpha\alpha})+\frac{\partial}{\partial\beta}(h_1h_3\sigma_{\alpha\beta})+\frac{\partial}{\partial\gamma}(h_1h_2\sigma_{\alpha\gamma})-\sigma_{\beta\beta}h_3\frac{\partial h_2}{\partial\alpha}-\sigma_{\gamma\gamma}h_2\frac{\partial h_3}{\partial\alpha}+\sigma_{\alpha\beta}h_3\frac{\partial h_1}{\partial\beta}+$$
$$+\sigma_{\alpha\gamma}h_2\frac{\partial h_1}{\partial\gamma}=\rho\frac{\partial^2u_\alpha}{\partial t^2};$$
$$\frac{\partial}{\partial\alpha}(h_2h_3\sigma_{\alpha\beta})+\frac{\partial}{\partial\beta}(h_1h_3\sigma_{\beta\beta})+\frac{\partial}{\partial\gamma}(h_1h_2\sigma_{\beta\gamma})-\sigma_{\gamma\gamma}h_1\frac{\partial h_3}{\partial\beta}-\sigma_{\alpha\alpha}h_3\frac{\partial h_1}{\partial\beta}+\sigma_{\beta\gamma}h_1\frac{\partial h_2}{\partial\gamma}+$$
$$+\sigma_{\alpha\beta}h_3\frac{\partial h_2}{\partial\alpha}=\rho\frac{\partial^2u_\beta}{\partial t^2};$$
$$\frac{\partial}{\partial\alpha}(h_2h_3\sigma_{\alpha\gamma})+\frac{\partial}{\partial\beta}(h_1h_3\sigma_{\beta\gamma})+\frac{\partial}{\partial\gamma}(h_1h_2\sigma_{\gamma\gamma})--\sigma_{\alpha\alpha}h_2\frac{\partial h_1}{\partial\gamma}-\sigma_{\beta\beta}h_1\frac{\partial h_2}{\partial\gamma}+\sigma_{\alpha\gamma}h_2\frac{\partial h_3}{\partial\alpha}+$$
$$+\sigma_{\beta\gamma}h_1\frac{\partial h_3}{\partial\beta}=\rho\frac{\partial^2u_\gamma}{\partial t^2};$$

(33)

where h_1,h_2,h_3 are the Láme coefficients:

$$h_j^2=\left(\frac{\partial\delta j}{\partial x}\right)^2+\left(\frac{\partial\delta j}{\partial y}\right)^2+\left(\frac{\partial\delta j}{\partial z}\right)^2;$$

$$\delta_1=\alpha;\qquad\delta_2=\beta;\qquad\delta_3=\gamma.$$

In Eq. (33), bulk forces are not taken into account.

In particular, in cylindrical coordinates, Eqs. (33) are rewritten as follows:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=\rho\frac{\partial^2u_r}{\partial t^2};$$
$$\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{\theta z}}{\partial z}+\frac{2\sigma_{r\theta}}{r}=\rho\frac{\partial^2u_\theta}{\partial t^2};\ (34)$$
$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta z}}{\partial\theta}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=\rho\frac{\partial^2u_z}{\partial t^2};$$

Equations (33) and (34) in displacements in the case of nonlinear function $\sigma_{ij}\sim\varepsilon_{ij}$ are very complicated. However, for linear function $\sigma_{ij}\sim\varepsilon_{ij}$, these equations are simplified by introducing potentials of longitudinal and transversal waves according to the following equation:

$$\vec{U}=grad\Phi+rot\vec{\Psi}, \qquad (35)$$

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herewith, the vector potential of transversal waves $\vec{\Psi}$ should satisfy the supplemental condition [10]:

$$div\vec{\Psi} = 0. \tag{36}$$

Equation (36) in arbitrary orthogonal coordinates is sufficiently complicated. In cylindrical coordinates (r,θ,z), Eq. (36) is satisfied automatically if the vector potential $\vec{\Psi}$ is set to:

$$\vec{\Psi} = \Psi_1 \vec{e_z} + rot(\Psi_1 \vec{e_z}). \tag{37}$$

In the potentials Φ and $\vec{\Psi}$, Eqs. (33) in linear form are as follows:

$$N_1(\Delta\Phi) = \frac{1}{a^2}\frac{\partial^2\Phi}{\partial t^2};$$
$$M_1(\Delta\vec{\Psi}) = \frac{1}{b^2}\frac{\partial^2\vec{\Psi}}{\partial t^2}, \tag{38}$$

where a and b are the propagation speeds of longitudinal and transversal waves, and the operators N and M are:

$$N_1 = R_0 + \frac{4}{3}R; \qquad M_1 = R. \tag{39}$$

In solid state dynamics, the stressed-strained state is uniquely determined by boundary and initial conditions.

In general case, the initial conditions are as follows:

$$\vec{U}\Big|_{t=0} = \vec{U}_1; \qquad \frac{\partial\vec{U}}{\partial t}\Big|_{t=0} = U_2. \tag{40}$$

In terms of boundary conditions, three main problems are highlighted.

The first problem takes place when at the boundary S the stresses are preset, applied to it in certain time. If projections of these forces related to unit surface area are denoted as X_n, Y_n, Z_n, then the boundary conditions are written as follows:

$$\sigma_{xx}\cos(n,x) + \sigma_{xy}\cos(n,y) + \sigma_{xz}\cos(n,z) = X_n; \tag{41}$$
$$\sigma_{xy}\cos(n,x) + \sigma_{yy}\cos(n,y) + \sigma_{yz}\cos(n,z) = Y_n;$$
$$\sigma_{xz}\cos(n,x) + \sigma_{yz}\cos(n,y) + \sigma_{zz}\cos(n,z) = Z_n,$$

where n is the normal to body surface S.

In the case of **the second problem,** displacements are preset on the surface S:

$$U_n = f_1(s_1,s_2); \qquad U_{s_1} = f_2(s_1,s_2); \qquad U_{s_2} = f_3(s_1,s_2), \tag{42}$$

where (n,s_1,s_2) is the orthogonal coordinate system on surface.

The third problem occurs in the case when on one part of the surface S the conditions (41) are preset, and on the remaining part – the conditions (42).

In addition to the main boundary problems, there are cases when continuous body is comprised of mediums with various properties. In this case, various conditions of contact can be preset on medium interface Γ.

1. In the case of absolutely rigid contact along the interface Γ, normal and tangential stresses are continuous, that is,

$$\sigma_{nn}^{(1)} = \sigma_{nn}^{(2)}; \sigma_{ns_1}^{(1)} = \sigma_{ns_1}^{(2)}; \sigma_{ns_2}^{(1)} = \sigma_{ns_2}^{(2)};$$
$$U_n^{(1)} = U_n^{(2)}; U_{s_1}^{(1)} = U_{s_1}^{(2)}; U_{s_2}^{(1)} = U_{s_2}^{(2)}. \tag{43}$$

2. If at the interface Γ there is no friction, then:

$$\sigma_{nn}^{(1)} = \sigma_{nn}^{(2)}; \sigma_{ns_1}^{(j)} = o_j (i,j=1,2); U_n^{(1)} = U_n^{(2)}. \tag{44}$$

3. In the case of friction along the contact Γ, its conditions are complicated and up till now there is no unique opinion concerning the type of these conditions. If one of the mediums is absolutely solid body, then such conditions can be presented by the Coulomb law.

The formulated main and supplemental boundary conditions occur generally upon solution of dynamic problems.

Now let us develop general oscillation equations of viscoelastic isotropic medium in linear approximation.

Isotropic viscoelastic medium of infinite sizes is considered in the plan as 3D viscoelastic body which at certain time $t = 0$ is impacted by external nonstationary forces applied to its surfaces and causing longitudinal or transversal oscillations.

Since plate oscillation is considered in linear approximation, then it would be more convenient to write the displacement equations in terms of

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potentials of longitudinal and transversal waves (38):

$$N(\Delta\Phi)=\rho\frac{\partial^2\Phi}{\partial t^2};\ M\left(\Delta\vec{\Psi}\right)=\rho\frac{\partial^2\vec{\Psi}}{\partial t^2}. \quad (45)$$

Herewith, Eq. (36) in Cartesian coordinates is as follows:

$$\frac{\partial\Psi_1}{\partial x}+\frac{\partial\Psi_2}{\partial y}+\frac{\partial\Psi_3}{\partial z}=0;$$

$$\vec{\Psi}=\vec{i}\,\Psi_1+\vec{j}\,\Psi_2+\vec{k}\,\Psi_3, \quad (46)$$

which imposes constraint on components of the vector potential $\vec{\Psi}$.

Displacements u,v,w in terms of the potentials Φ and ψ_j are expressed by the equations:

$$u=\frac{\partial\Phi}{\partial x}+\frac{\partial\psi_3}{\partial y}-\frac{\partial\psi_2}{\partial z};$$
$$v=\frac{\partial\Phi}{\partial y}+\frac{\partial\psi_1}{\partial z}-\frac{\partial\psi_3}{\partial x};$$
$$w=\frac{\partial\Phi}{\partial z}+\frac{\partial\psi_2}{\partial x}-\frac{\partial\psi_1}{\partial y}. \quad (47)$$

Similarly, the stresses σ_{ij} in terms of Φ and Ψ_j are expressed as follows:

$$\sigma_{xx}=L(\Delta\Phi)+2M\left(\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\psi_3}{\partial x\partial y}-\frac{\partial^2\psi_2}{\partial x\partial z}\right);$$
$$\sigma_{yy}=L(\Delta\Phi)+2M\left(\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\psi_1}{\partial y\partial z}-\frac{\partial^2\psi_3}{\partial x\partial y}\right); \quad (48)$$
$$\sigma_{zz}=L(\Delta\Phi)+2M\left(\frac{\partial^2\Phi}{\partial z^2}+\frac{\partial^2\psi_2}{\partial x\partial z}-\frac{\partial^2\psi_1}{\partial y\partial z}\right);$$
$$\sigma_{xy}=M\left(2\frac{\partial^2\Phi}{\partial x\partial y}+\frac{\partial^2\psi_1}{\partial x\partial z}-\frac{\partial^2\psi_2}{\partial y\partial z}-\frac{\partial^2\psi_3}{\partial x^2}+\frac{\partial^2\psi_3}{\partial y^2}\right);$$
$$\sigma_{yz}=M\left(2\frac{\partial^2\Phi}{\partial y\partial z}-\frac{\partial^2\psi_1}{\partial y^2}+\frac{\partial^2\psi_1}{\partial z^2}+\frac{\partial^2\psi_2}{\partial x\partial y}-\frac{\partial^2\psi_3}{\partial x\partial z}\right);$$
$$\sigma_{xz}=M\left(2\frac{\partial^2\Phi}{\partial x\partial z}-\frac{\partial^2\psi_1}{\partial x\partial y}+\frac{\partial^2\psi_2}{\partial x^2}-\frac{\partial^2\psi_2}{\partial z^2}+\frac{\partial^2\psi_3}{\partial y\partial z}\right).$$

where $L=N-2M$; $M=GR$;

$$N=KR_0+\frac{4}{3}GR.$$

Oscillations of viscoelastic plate in general case are caused by forces applied to the surfaces $z=\pm h$, that is, the boundary conditions are:

$$\sigma_{zz}=F_z^{\pm}(x,y,t);\ \sigma_{xz}=F_{xz}^{\pm}(x,y,t);$$
$$\sigma_{yz}=F_{yz}^{\pm}(x,y,t);\ z=\pm h. \quad (49)$$

Let us consider that initial conditions are zero:

$$\Phi=\psi_j=\frac{\partial\Phi}{\partial t}=\frac{\partial\psi}{\partial t}=0,\ t=0. \quad (50)$$

that is, the three-layer shells at $t<0$ are in the state of rest.

Let us substitute external conditions in the following form [11]:

$$F_z^{\pm}=\int_0^{\infty}\left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}dq\int_l f_{z,0}^{\pm}e^{Pt}dp;$$
$$F_{xz}^{\pm}=\int_0^{\infty}\left.\begin{matrix}\cos kx\\\sin kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}dq\int_l f_{xz,0}^{\pm}e^{Pt}dp; \quad (51)$$
$$F_{yz}^{\pm}=\int_0^{\infty}\left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\cos qy\\\sin qy\end{matrix}\right\}dq\int_l f_{yz,0}^{\pm}e^{Pt}dp,$$

and let us consider the functions $f_{z,0}^{\pm}$; $f_{xz,0}^{\pm}$; $f_{yz,0}^{\pm}$ negligible outside of the region:

$$|k|\le k_0;\ |J_{mp}|\le\omega_0;\ |q|\le q_0, \quad (52)$$

that is, the external conditions do not contain high frequency harmonics, otherwise, the length of propagating waves both in time and coordinate exceeds transversal sizes of plate, which is obvious in terms of physics, otherwise, the plate as 3D body cannot be presented as 2D body.

Equations (45)–(50) will be solved with the following assumptions:

$$\Phi=\int_0^{\infty}\left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}dq\int_l\Phi_0e^{Pt}dp;$$
$$\psi_1=\int_0^{\infty}\left.\begin{matrix}\sin kx\\-\cos kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\cos qy\\\sin qy\end{matrix}\right\}dq\int_l\psi_{10}e^{Pt}dp;$$
$$\psi_2=\int_0^{\infty}\left.\begin{matrix}\cos kx\\\sin kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\sin qy\\-\cos qy\end{matrix}\right\}dq\int_l\psi_{20}e^{Pt}dp; \quad (53)$$
$$\psi_3=\int_0^{\infty}\left.\begin{matrix}\cos kx\\\sin kx\end{matrix}\right\}dk\int_0^{\infty}\left.\begin{matrix}\cos qy\\\sin qy\end{matrix}\right\}dq\int_l\psi_{30}e^{Pt}dp.$$

Equation (52) allows to strictly differentiate Eq. (53) with regard to coordinates and time and to substitute them into Eq. (45), the boundary and initial conditions (49) and (50). Then, in order to determine Φ_0 and ψ_{j0}, we have the following equations:

$$\frac{d^2\Phi_0}{dz^2}-\alpha^2\Phi_0=0;$$
$$\frac{d^2\psi_{j0}}{dz^2}-\beta^2\psi_{j0}=0, \qquad (54)$$

where

$$\alpha^2=k^2+q^2+\frac{\rho p^2}{N_0};$$
$$\beta^2=k^2+q^2+\frac{\rho p^2}{M_0}; \qquad (55)$$

N_0 and M_0 are the Laplace transformed operators N and M.

The transformed displacements and stresses in terms of Φ_0 and ψ_{j0} are as follows:

$$u_0=k\Phi_0-\frac{d\psi_{20}}{dz}-q\psi_{30};$$
$$v_0=q\Phi_0+\frac{d\psi_{10}}{dz}+k\psi_{30};$$
$$w_0=\frac{d\Phi_0}{dz}+q\psi_{10}-k\psi_{20}; \qquad (56)$$
$$\sigma_{xx}^{(0)}=L_0\left[\frac{d^2\Phi_0}{dz_2}-\left(k^2+q^2\right)\Phi_0\right]+2M_0\left[-k^2\Phi_0+k\frac{d\psi_{20}}{dz}+kq\psi_{30}\right];$$
$$\sigma_{yy}^{(0)}=L_0\left[\frac{d^2\Phi_0}{dz_2}-\left(k^2+q^2\right)\Phi_0\right]+2M_0\left[-q^2\Phi_0+q\frac{d\psi_{10}}{dz}-kq\psi_{30}\right]; \qquad (57)$$
$$\sigma_{zz}^{(0)}=L_0\left[\frac{d^2\Phi_0}{dz_2}-\left(k^2+q^2\right)\Phi_0\right]+2M_0\left[\frac{d^2\Phi_0}{dz_2}+q\frac{d\psi_{10}}{dz}-k\frac{d\psi_{20}}{dz}\right];$$
$$\sigma_{xy}^{(0)}=M_0\left[2kq\Phi_0+k\frac{d\psi_{10}}{dz}-q\frac{d\psi_{20}}{dz}+\left(k^2-q^2\right)\psi_{30}\right];$$
$$\sigma_{yz}^{(0)}=M_0\left[2q\frac{d\Phi_0}{dz}+q^2\psi_{10}+\frac{d^2\psi_{10}}{dz^2}-kq\psi_{20}+k\frac{d\psi_{30}}{dz}\right];$$
$$\sigma_{xz}^{(0)}=M_0\left[2k\frac{d\Phi_0}{dz}+kq\psi_{10}-k^2\psi_{20}-\frac{d^2\psi_{20}}{dz^2}-q\frac{d\psi_{30}}{dz}\right];$$

General solutions of Eq. (54) are as follows:

$$\Phi_0=A_1ch(\alpha z)+A_2sh(\alpha z);$$
$$\psi_{10}=B_{11}sh(\beta z)+B_{12}ch(\beta z);$$
$$\psi_{20}=B_{21}sh(\beta z)+B_{22}ch(\beta z);$$
$$\psi_{30}=B_{31}ch(\beta z)+B_{32}sh(\beta z); \qquad (58)$$

herewith, the integration constants B_{ij} due to Eq. (46) are interrelated as follows:

$$kB_{1i}+qB_{2i}+\beta B_{3i}=0,\ i=1;2. \qquad (59)$$

In order to determine the integration constants A_i, B_{ij}, we have the boundary conditions (49) which in the transformed potentials Φ_0 and ψ_{j0} are as follows:

$$L_0\left[\frac{d^2\Phi_0}{dz^2}-\left(k^2+q^2\right)\Phi_0\right]+2M_0\left[\frac{d^2\Phi_0}{dz^2}+q\frac{d\psi_{10}}{dz}-k\frac{d\psi_{20}}{dz}\right]=f_{z,0}^{\pm};$$
$$M_0\left[2k\frac{d\Phi_0}{dz}+kq\psi_{10}-k^2\psi_{20}-\frac{d^2\psi_{20}}{dz^2}-q\frac{d\psi_{30}}{dz}\right]=f_{xz,0}^{\pm}; \qquad (60)$$
$$M_0\left[2q\frac{d\Phi_0}{dz}+q^2\psi_{10}+\frac{d^2\psi_{10}}{dz^2}-kq\psi_{20}+k\frac{d\psi_{30}}{dz}\right]=f_{xz,0}^{\pm},$$

Since the problem is linear, let us consider separately longitudinal and transversal oscillations of viscoelastic plate [12].

Analysis of wave processes in constrained deformed bodies is reduced to complex mathematical problems, which at present can be solved neither by analytical nor by numerical methods.

Even for deformed mediums described by the simplest models, such as elastic and viscoelastic mediums, numerous nonstationary problems are not solved and there are no methods capable to solve these problems in exact formulation. Therefore, many application problems are solved on the basis of simplified models reducing spatial problems of dynamics to two- or one-dimensional cases. Such simplified models are presented by plates, rods, and shells.

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4. CONCLUSION

Simplified model is applied for studying nonstationary oscillations of plates, rods, and shells, motion of such systems are simulated by motion of points of center plane (plate), center surface (shell), center line (rods).

Classical theories of oscillation of rods and plates are based on hypotheses of plane cross section and the Kirchhoff hypothesis. On the basis of these hypotheses, approximate oscillation equations are derived known as classical equations.

Sharp increase in the number of application problems leading to analysis of dynamic behavior of rods, plates, and shells under various nonstationary external impacts revealed insufficiency of the classical equations for description of the considered phenomena, in its turn this resulted in high number of various updated oscillation theories and equations. These updated equations are also based on new hypotheses of various researchers.

It should be mentioned that there are few works devoted to approximate oscillation equations of rods and plates with consideration for more complicated mechanical and rheological properties of rod or plate material, nonlinear dependence of stresses on deformations, consideration for initial shifts and stresses with accounting for ambient temperature and environment, anisotropy, etc.

Non-unique approach to derivation of oscillation equations of rods, plates, and shells is attributed to the fact that these equations were derived on the basis of various hypotheses aimed at approximate description of distribution of required shifts and stresses through cross sections of these bodies. Various approximate oscillation equations are based on these hypotheses.

However, analysis of oscillations of rods, plates, and shells can be based on accurate formulation of the problem for rods, plates, and shells as 3D bodies under external impacts leading to this or that oscillation type.

While solving these 3D problems, it is possible to obtain with this or that degree of accuracy the oscillation equations of these bodies depending on external conditions, fixing conditions of interfaces with consideration for certain mechanical, rheological and other properties of material.

Such mathematical approach was applied by Vlasov for determination of stress and strain state of elastic isotropic homogeneous plates in linear formulation under stationary ambient conditions. This approach is known as the method of initial functions.

This work attempted to apply the mathematical theory of oscillations of elastic or viscoelastic plate to analysis of their dynamic behavior under nonstationary external impacts. On the basis of mathematical approach, exact equations of longitudinal and transversal oscillations of viscous plates with and without consideration for initial shifts and stresses as well as approximate equations with accounting for ambient environment and physical nonlinearity of material have been obtained.

For all problems, the equations are obtained for all shifts and stresses across plate thickness, the main boundary problems are formulated leading to longitudinal or transversal oscillation of plate.

On the basis of the exact equations, certain resulting approximate equations have been analyzed, and approximate boundary conditions have been formulated.

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