Journal of Theoretical and Applied Information Technology
15th August 2021. Vol.99. No 15

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A NOVEL APPROACH BASED ON ACTIVE CONSTRAINT FOR MINIMIZING VAR IN THE PORTOFOLIO OPTIMIZATION PROBLEM

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ABSTRACT

The portfolio optimization mathematical model is more frequently expressed with an eye to minimize the Value at Risk (VaR). Markowitz's difficulties in managing the quadratic programming model were alleviated by recent advances in algorithmic analysis, which sparked interest in overcoming real constraints in portfolio selection by introducing a linear risk function. The point of this article is to address the issue of portfolio selection of minimum transaction lots. An improved search algorithm appertaining to active constraints is presented to interpret the integer programming model. The algorithm leads by solving the relaxed problem in order to reach a settlement that is similar to a continuous solution.

Keywords: *Mixed Integer Programming, Portfolio Optimization, Active Constrained*

1. INTRODUCTION

Investment is an activity of investing a number of funds in the form of money or goods which are expected to give more results in the future. This activity takes the form of buying securities (securities) which is usually carried out through the money market or the capital market. The investment instruments that are invested in the money market are deposits, Indonesian bank certificates (SBI) and foreign currency, while those that are invested in the capital market involving stocks, bonds, mutual funds, exchange traded funds (ETF) and derivatives. According to Jones [1], an investment is defined as an asset contribution within one or more properties owned in relation to the use of financial assets, such as deposit certificates, bonds, stocks, or mutual funds, over a long period of time.

Medium and long-term investment management problems are frequently conceived as a dynamic portfolio selection problem, where investment decisions are, for example, changing from time to time. Such options become standard given the time constraints and dependencies of the countries involved. As a result of market frictions including trade and regulatory expenses, as well as tax limitations, challenges are becoming more dynamic, discrete, multi-stage, and control-related. The nature of the risk measures used over multiple time periods is critical to achieving successful risk control. When compared to static conditions, it is difficult to construct adequate risk measurements of a multi-period nature that are realistic and realistically meaningful [2].

If an investor wants to maximize his or her returns, he or she must devise a good strategy. To reduce investment risk, investors can diversify, which is done by combining various securities in their investments, forming portfolios. And, if an investor desire to build a portfolio, he or she must be able to properly analyze the current market. The intended portfolio is a collection of documents on a person's, group's, institution's, organization's, company's, or the like's assets that aims to document the progress of a process in achieving predetermined goals. There are portfolios in a portfolio that are not limited in number or a lot, and in the formation of the portfolio, the investor will choose the right one from the many existing portfolios, resulting in the optimal portfolio. This is an optimization problem with a mathematical model. The goal of optimization is to find a series of portfolios with the lowest risk level for each specific rate of return or, alternatively, the highest rate of return for each specific risk. The portfolio optimization problem's main goal is to find the optimal portfolio with the lowest variance of all possible portfolio sets for any expected rate of return.

In risk measurement, variance is a tool that plays an important role in optimizing under uncertainty, particularly when dealing with investment losses. The variance is denoted by the symbol VaR. VaR is non-convex and combinatorial, according to Gaivoronski and Pflug [3]. This makes the VaR portfolio problem fundamentally complicated to sort out [4], but in [5], VaR is a quantifiable risk measure. The mean is used to measure the return and is associated to degree of precision the risk measure can be estimated.

The VaR and the mean are two measuring instruments used in portfolio selection. The initial mathematical formulation of the portfolio selection problem was developed by Markowitz [6]. The portfolio selection problem was developed by Markowitz as a tradeoff linking the mean and variance of the asset portfolio, which is known as the mean-variance (MV) model. Maintain constant variance and maximize expected return, or maintain constant variance and minimize return to streamline the portfolio so that investors can choose a portfolio mix based on their risk. Markowitz variant optimization is a well-known investment theory that is widely used in asset allocation. The most significant impact can be seen in portfolio management practices. This theory is concerned with the assessment and management of risk and the return on portfolio investment. This is very advantageous because the resulting portfolio optimization will have the same expected return with less risk or higher expected returns with the same level of risk.

Despite that, there are some disadvantages or limitations to Mean-Variance (MV), such as parameter uncertainty, which is a significant issue in optimization problems. The uncertainty in market parameters affects the best approach of the problem in the Markowitz model, so the results are unreliable and the computational complexity is high, and the input problem is required for calculations where an investor obtains the estimate (return, variance, and covariance) for each stock / securities included in the portfolio. If the portfolio contains N assets, an estimate of N returns, an estimate of N variance, and an estimate of the covariance of $N\ (N\text{-}1)\ /\ 2$ are required, and the result is $2N + N\ (N - 1)\ /\ 2$, so that as the number of assets in the portfolio grows, so does the total required parameter/estimate, because the covariance between each asset must be estimated [7].

2. RELATED WORKS

The Markowitz model's limitations prompted the development of new theories in portfolio problems. Konno and Yamazaki [8] developed the earliest linear model for portfolio selection. This model is in the form of linear programming, making it easier to use for optimizing large portfolios. Furthermore, because this model does not need to calculate the correlation and covariance of each asset return, the computation process is faster and more efficient. The linear version of this model employs a risk function distinct from the classic portfolio variant, more particularly the absolute deviation portfolio. A noteworthy hallmark of this model is that no probabilistic approximations are created on the security level of returns, whereas the multivariate normal distribution model is demonstrated to be indistinguishable to the Markowitz model as provision for the rate of return.

Mansini [9], in addition to Konno and Yamazaki, created a portfolio optimization optimization model using a linear program, which resulted in portfolio optimization using a linear program and demonstrated that LP (Linear Programming) was more reliable than QP (Quadratic Programming).

Then Zenios and Kang [10] developed the Konno and Yamazaki model, which yielded the Mean Absolute Deviation (MAD) model for mortgage-backed portfolio optimization securities by demonstrating how a suitable choice of coefficients in linear combinations yielded a model identical to Konno and Yamazaki, but with half the number of constraints. The MAD model is also regarded as a viable alternative to the traditional MV model because it considers absolute mean deviations rather than standard deviations [11]. Feinstein and Thapa [12] independently obtained comparable results, namely that, some of assets in the best-performing portfolio with no non-zero asset limits (assuming no upper bound) is at most $2T + 2$, where T is the number of time periods for estimating profit distribution asset parameters. The amount of non-zero assets in the portfolio is optimal in this formulation.

Furthermore, Zymler [13] contends that portfolio optimization responds to investment uncertainty by selecting a portfolio that maximizes profits while achieving a certain level of risk that is calculated or, more accurately, minimizes variance, with the constraints obtained at a predetermined rate of return.

Furthermore, Valle [14] presents three portfolio optimization models for three distinct problems. The first issue concerns the selection of an Absolute Return Portfolio (ARP). ARP is commonly regarded as a financial portfolio designed to generate high returns nonetheless of how the fundamental market enact. The presented program is a three-stage zero-integer solution to the ARP selection problem. The second issue to consider is creating a Market Neutral Portfolio (MNP).

MNP is generally defined as a financial portfolio that (ideally) outperforms the underlying market. MNP was designed with mixed-integer nonlinear programming (MINLP) in mind, with the goal of minimizing the absolute value of the correlation between portfolio returns and underlying benchmark returns. The third issue is about Exchange-Traded Funds (ETFs). ETFs are open-market funds whose performance is typically linked to a benchmark index. Performance studies are also discussed in depth in the ETF market, with the conclusion that ETF performance is consistently low. More than just index tracking. Furthermore, MINLP is presented on the topic of selecting assets that comprise ETFs.

Hosseinia and Hamidi [15], unlike previous researchers, developed a general fund investment portfolio optimization model using a fuzzy approach. Houda [16] presents two numerical methods of mathematical optimization problems for one and multiple purposes (ILP, IGP), with two values (0 and 1) as decision variables.

Rankovic [17] repurposed Markowitz's measuring instruments in 2016. Rankovic models the portfolio optimization problem with the new mean Value at Risk (VaR) optimization method, with VaR approximated using the univariate Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) volatility model.

Rahnama [18] conducted a study in the same year that focused on the Markowitz MV portfolio optimization optimization problem with cardinality constraints and the dependent variable called the modern portfolio optimization problem, which is a MINLP problem and is known as the NP-Hard problem. Due to the complexity of the covariance matrix structure, precise methods such as branching and cutting, even when resolved by CPLEX, cannot solve large samples in a reasonable time.

In other words, as more securities are added to the portfolio, the calculation grows geometrically because the number of correlation coefficients considered for the covariance matrix is (n-1) / 2 independent entries, requiring a large number of combinations to be calculated in order to select the non-well-correlated assets from the matrix covariance. The complexity of portfolio selection problems, as well as the need to select optimal portfolios in a reasonable time in the real stock market where transactions must be quick, necessitate efficient methods to interpret portfolio selection problems that take into account the trade-off between solution quality and computation time, which is the goal of this study.

A foundational assumption on Markowitz model is that the allocation of the assets return is normal [19]. However, soon later, Mandelbrot [20] and Fama [21] suggested that this distribution would not be normal in 10 general. It is also further certified empirically that asset returns exhibit fat tails and asymmetry [22]. Besides risks measured by variance, there are risks controlled by skewness and kurtosis as well [23]. Therefore, generalized Markowitz models taking into account skewness or (and) kurtosis have been becoming a research frontier, see [24]–[27] and references therein. As variance is related to a quadratic polynomial, the mean-variance model gives a quadratic programming [28]. Likewise, a generalized Markowitz model involving skewness and kurtosis will give a polynomial optimization problem involving cubic and quartic polynomials [27]. In general, a polynomial optimization problem is nonconvex and NP-hard to solve [29]. Thus, global optimal solutions to the proposed generalized Markowitz models cannot be found with a guarantee, and in turn it becomes subtle to either compare or give a conclusion for these models precisely. This notorious property of general polynomial optimization 20 problems therefore becomes a huge obstacle for applications of generalized Markowitz models. Nevertheless, various approaches have been proposed for solving generalized Markowitz models, see [27], [28], [30]–[33].

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Using previous research findings, the author conducted a study utilizing an alternative method to optimize the portfolio, namely the active constraint strategy or method. The MV model has never been used in conjunction with this method before. This is why the MV model was chosen as one of the variables in this study.

According to Chinneck [34], active constraints include all equality constraints and inequalities at the point of the equation. This definition includes all inequalities between active constraints; it is related to the nonbasis variable. The optimal point will yield a solution. Further investigation will reveal the location of the test point and vice versa. This satisfies the requirement that the majority of the variables influence the active constraint at the LP point of optimum relaxation. The active constraint strategy, according to Mawengkang [35], is a strategy for searching for global optimal solutions by removing non-base variables from its boundaries so that the basic feasible solution is close to its limit and combined with the concept of superbasis variables, namely variables that are not at the limit. The active constraint method has the advantage of taking returns into account when determining risk.

Active constraints, according to these definitions, are constraints that form an extreme point. Inactive constraints, on the other hand, are constraints that do not form an extreme point. Redundancy occurs when a constraint does not determine which part of the feasible area it is in. A feasible solution to an optimization problem is the set of values for the decision variables that satisfy the constraints at the same time. The existing constraints define the solution's feasible region. The best interpretation is a set of decision variable values that satisfy all of the constraints.

There are two methods for optimizing the model: linear programming and non-linear programming. Using the amount of information available, the linear programming method can be used to clearly formulate problems. After the problem has been thoroughly defined, the next step is to convert it into a mathematical model.

Minimize: $f(\vec{x})$

$$\vec{x} = (x_1, \cdots, x_n) \in R^n$$

$$l_i \leq x_i \leq u_i \qquad i = a, \dots, n$$

Constraint: $g_j(\vec{x}) \leq 0 \quad j = i, \dots, q$

$$h_j(\vec{x}) = 0 \quad j = q+1, \dots, m$$

The search space, S, is identified as an n-dimensional rectangle in $R^n (S \subseteq R^n)$, and the objective function f is defined on it. A lower and upper bound define the variable domain. An additional set of constraints $m (m \geq 0)$ defines a feasible area $F \subseteq S$, and $\vec{x}$ is defined on a feasible space ($\vec{x} \in F \in S$). The active constraint at $\vec{x}$ is the boundary g_j satisfying $g_j(\vec{x}) = 0$ at any point $\vec{x} \in F$. As a result, the equality constraint h_j is active at all S points [36].

Active constraint strategies are being researched. Mawengkang [35] efficiently solves non-linear mathematical programming problems by combining the active constraint method with non-basis variables. After ignoring the integral requirement and solving the problem, this strategy is accustomed to oblige the correct non-integer base variable to shift to a point integer neighborhood.

Erwin [37] discusses nonlinear integer programming problems that are large-scale, highly combinatorial, and highly nonlinear. This problem is structured by a subset of variables bounded to undertake discrete, linear values that can be separated from continuous variables by providing a direct search method to achieve integer feasibility for a class of mixed non-linear programming problems in a moderately short time. The direct search method combines the active constraint method and the concept of a superbase with a strategy of removing non-basic variables from constraints.

Mansyur [38] conducted another study on a subset of the nonlinear mathematical programming problems discussed in this study. To efficiently solve problems, a technique has been devised to free non-basic variables from their constraints in conjunction with the active restriction approach and the concept of the magnificent foundation. Following ignorance of the integral requirement and resolution of the problem, this method is applied to drive the appropriate non-integer basis variables into their integer point environment.

Tambunan [39] also presents a solution based on active constraints for dealing with specific MINLP classes. The variable of the superbasis idea that is not closely bound to obtain the active constraint from the nonlinear objective and the constraint function is the variable that is not closely bound to obtain the active constraint from the nonlinear objective and the constraint function. Active constraints are used to determine a global optimal point where a feasible solution approaches its limit.

Sitopu[40] discusses nonlinear mathematical programming problems that are specialized in structures distinguished by a subset of variables that are limited to assuming discrete values, are linear, and can be separated from continuous variables in another study. Combining the active constraint method with a strategy for releasing non-basis variables from the limit has resulted in the development of a strategy for releasing non-basis variables from the limit. This procedure is used to coerce the corresponding non-integer base variable into the integer point environment.

3. PORTFOLIO OPTIMIZATION MODEL

The investor utility function, which serves as the foundation aimed at reducing risk factor tolerance, is discussed first in this mathematical model. Second, without risk-free assets, investment portfolio modeling discusses the Mean-VaR.

3.1 Investor Utility Function

Individual investors generally have different equations of curves or utility functions based on their risk tolerance in investing. The investor's risk avoidance function can be determined based on a person's utility function. The utility functions discussed here are in square form.

Let W represent the investment property (funds). Assume that an investor also has a rectangular utility function, as shown below [41], [42]:

$U(W) = W - bW^2$; with parameter coefficient $b > 0$.

This utility function's first and second derivatives are as follows:

$U'(W) = 1 - 2bW > 0$ for $W < 1/2b$ and $U''(W) = -2b < 0$

One of these investors' risk aversion function $\eta(W)$ can be defined as follows:

$$\eta(W) = \frac{U''(W)}{U'(W)} = \frac{-2b}{1-2bW} = \frac{2b}{1-2bW}$$

As a result, the risk tolerance factor τ can be calculated as follows:

$$\tau(W) = \frac{1}{\eta(W)} = \frac{1-2bW}{2b}$$

If the initial funds are invested $W = W_0$, the following variables become risk tolerance:

$$\tau(W_0) = \frac{1}{\eta(W_0)} = \frac{1-2bW_0}{2b}$$

The risk tolerance factor, which is then used to create an optimal portfolio in accordance with Value-at-Risk, is discussed further below.

3.2 Modeling Mean-VaR Portfolio Optimization

Discrete time portfolios are mentioned in this section. Assume the return on asset i where $i = 1,2,\ldots,N$ and N is the total number of assets in the portfolio. The portfolio return is the average weighted return of the portfolio's assets [43]. If the investor selects a portfolio with the following vector weights:

$$w^T = (w_1, \ldots, w_N), \sum_{i=1}^{N} w_i = 1$$

where w_i is the proportion (weight) of funds invested in asset i, then the vector portfolio weight w, r_p return is provided by [44]:

$$r_p = \sum_{i=1}^{N} w_i r_i \quad (1)$$

The average portfolio (expected return) μ_p is given by equation (1):

$$\mu_p = E[r_p] \sum_{i=1}^{N} w_i E[r_i] = \sum_{i=1}^{N} w_i \mu_i \quad (2)$$

And the σ_p^2 portfolio variance is calculated as follows:

$$\sigma_p^2 = \text{Var } r_p = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \text{Cov } (r_i, r_j) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \quad (3)$$

The σ_{ij} expresses the covariance between i and j shares. That is, it is written as follows:

$$\sigma_{ij} = \text{Cov } (r_i, r_j) = E[(r_i - \mu_i)(r_j - \mu_j)] = \rho_{ij}\sigma_i\sigma_j$$

Journal of Theoretical and Applied Information Technology
15th August 2021. Vol.99. No 15

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(4)

Where ρ_{ij} is the correlation coefficient between asset returns i and j, and $\sigma_i = \sqrt{\sigma_i^2}$ is the standard deviation of asset returns i[45].

Assume the Σ covariance matrix and the identity matrix are as follows:

$$\Sigma = \begin{pmatrix} & \sigma_1^1\sigma_{12}\ldots\sigma_{1N} & \\ & \sigma_{21}\sigma_2^2\ldots\sigma_{2N} & \\ \vdots & \vdots \quad \vdots & \vdots \\ & \sigma_{N1}\sigma_{N2}\ldots\sigma_N^2 & \end{pmatrix} \text{ and}$$

$$I = \begin{pmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}$$

where $\sigma_{ii} = \sigma_i^2$ with $i = 1, \ldots, N$

Furthermore, the vector equation can be used to express the expected portfolio returns in equation (2) as follows:

$$\mu_p = E[r_p] = w^T\mu \qquad (5)$$

and the variance equation (4.2.3) is as follows:

$$\sigma_p^2 = \text{Var}\,(r_p) = w^T\Sigma w \qquad (6)$$

According to [46], [47], the risk measurement model is Value-at-Risk for Portfolio p, expressed as $VaR_p = -W_0\{\mu_p + z_\alpha\sigma_p\}$. The Value-at-Risk for portfolio p can be demonstrated using equations (5) and (6) as follows:

$$VaR_p = -W_0\{\mathrm{w}^T\mu + z_\alpha(\mathrm{w}^T\Sigma\mathrm{w})^{1/2}\} \qquad (7)$$

Where the sign (-) portrays a loss, W_0 represents the initial capital invested, and z_α represents the percentile of the standard normal distribution when the level of significance is given $(1-\alpha)\%$.

Furthermore, an efficient portfolio is defined as follows:

Definition 1[48].

If there is no p portfolio with $\mu_p \geq \mu_{p^*}$ and $VaR_p < VaR_{p^*}$, the p^* portfolio is called (Mean - VaR) efficient.

Thus, if the risk of portfolio investment is measured using Value-at-Risk, then the investment portfolio optimization problem will be solved using Markowitz's, resulting in [49]:

Maximum $\{2\tau\mu_p - VaR_p\}$

Constraint $\sum_{i=1}^{N} w_i = 1$

If the initial investment is $W_0 = 1$ unit of money, the objective function is as follows:

Given $VaR_p = -W_0\{\mu_p + z_\alpha\sigma_p\}$ then,

$2\tau\mu_p - VaR_p$

$2\tau\mu_p + W_0\{\mu_p + z_\alpha\sigma_p\}$

$2\tau\mu_p + W_0\mu_p + W_0 z_\alpha\sigma_p$

Because $W_0 = 1$ unit of money, so

$2\tau\mu_p + \mu_p + z_\alpha\sigma_p$

$(2\tau + 1)\mu_p + z_\alpha\sigma_p$

It is then obtained.

Maximum $\{(2\tau + 1)\mu_p + z_\alpha\sigma_p\}$

Constraint $\sum_{i=1}^{N} w_i = 1$ (8)

with τ risk tolerance factor owned by investors. Because $\mu_p = \mathrm{w}^T\mu$ and $\sigma_p^2 = \mathrm{w}^T\Sigma\mathrm{w}$ Equation (8) can be expressed as follows in linear algebraic form:

$(2\tau + 1)\mu_p + z_\alpha\sigma_p$ as well as

Maximum $(2\tau + 1)\mathrm{w}^T\mu + z_\alpha(\mathrm{w}^T\Sigma\mathrm{w})^{1/2}$

Constraint $\mathrm{w}^T e = 1$ (9)

The Mean-VaR portfolio investment optimization problem is represented by equations (8) and (9).

The active constraint method is used in this study to find a solution to the Mean-VaR portfolio investment optimization question.

4. METHODS FOR OPTIMIZATION BASED ON ACTIVE CONSTRAINTS

This research looked at a class of algorithms wherein the search direction along the active constraint coat is defined as being between an orthogonal Z matrix and a normal constraint matrix. As a result, if $\hat{A}x = \hat{b}$ is the latest set of active

Journal of Theoretical and Applied Information Technology
15th August 2021. Vol.99. No 15

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

constraints $n-s$, Z is a $n \times s$ matrix that looks like this:

$$\hat{A}Z = 0 \quad (10)$$

The following are the main steps that must be completed in each iteration (by producing a proper descent direction, p):

1. Determine the reduced gradient $g_A = Z^T g$.
2. Create some approximations for the Hessian reduction, specifically $G_A \doteqdot Z^T GZ$.
3. Obtain approximations for systems of equations:
   $Z^T GZp_A = -Z^T g$ (11)
   by resolving the system
   $G_A p_A = -g_A$
4. Determine the direction to obtain $p = Zp_A$.
5. Use a line search to find the closest approximation to α^* where

$$f(\boldsymbol{x}+\alpha^* \boldsymbol{p}) = \min_{\substack{\alpha \\ \{x+\alpha p \text{ feasibel}\}}} f(\boldsymbol{x}+\alpha \boldsymbol{p})$$

In addition to having full column rankings, e.g. (10) is the only (algebraically) constraint on Z, and thus Z can be a couple forms. The Z parallel to the procedure itself, in particular, takes the form

$$Z = \begin{bmatrix} -W \\ I \\ 0 \end{bmatrix} = \begin{bmatrix} -b^{-1}S \\ I \\ 0 \end{bmatrix} \begin{matrix} \}m \\ \}s \\ \}n-m-s \end{matrix} \quad (12)$$

This is a simple description that will be used for exposition in the subsequent segment, but it should be indicated that it only works computationally with the S and triangular (LU) factorizations of B. The Z matrix is certainly not calculated in its entirety.

Z, whose column is orthonormal ($Z^T Z = I$), is recommended for good reason. The main advantage of the Z transformation is that it does not initiate redundant condition into the problem reduction (see steps A–D aforementioned, specifically equation (11)). This method has been used in programs in which Z is definitely accumulated as a dense matrix. The LDV factorization of the matrix $[B \quad S]$ allows for the expansion to the expansively scattered / sparse linear constraints:

$$[B \quad S] = [L \quad O]DV$$

where L is a triangle, D is a diagonal, and $D^{1/2}V$ is normal, and L and V are accumulated as products. Despite that, if S has numerous columns, this factorization will always be much denser than B's LU factorization. It is thus based on performances by continuing with Z in (12). Simultaneously, be aware (due to B^{-1}'s unwelcome appearance) that B must be cared for as best as possible.

Summary of the procedure:

This section provides an overview of the optimization algorithm.

Assume you have the following items:

1. $[B \quad S \quad N]x = b,\ l \le x \le u$ is satisfied by a viable vector x.
2. The equivalent function value $f(x)$ and the gradient vector $g(x) = [g_B \quad g_S \quad g_N]^T$.
3. The number of superbasis variables, $s(0 \le s \le n-m)$.
4. Factorization, LU, on the base matrix $B\ \ m \times m$.
5. The factorization, RTR, of the quasi-Newton approach to the $s \times s$ matrix is $Z^T GZ$. (It should be noted that G, Z, and $Z^T GZ$ are never truly counted).
6. A vector rr that meets $B^T \pi = g_B$.
7. The reduced gradient vector $h = g_S - S^T \pi$.
8. TOLRG and TOLDJ both have small positive convergence tolerances.

The portfolio model to be solved is derived from equation (9), namely

Maximum $\{(2\tau+1)w^T\mu + z_\alpha (w^T \Sigma w)^{1/2}\}$

Constraint $w^T e = 1$

This model contains a non-linear form of the objective function, as has been pointed out.

The Generalized Reduced Gradient method is used to solve the model by first using the Langrange function and then continuing as stated in the algorithm.

The algorithm will then proceed as follows:

Step 1. (Convergence testing in a known subspace). If $||h|| > \text{TOLRG}$, proceed to step 3.

Step 2. ("PRICE", i.e., calculate the Lagrange multiplier, add one superbase).

(a) Determine $\lambda = g_N - N^T\pi$.

(b) Choose

$\lambda_{q_1} < -\text{TOLDJ}\left(\lambda_{q_2} > +\text{TOLDJ}\right)$, the λ's largest element that corresponds to the variables in its upper (lower) bound. If not, STOP; Kuhn-Tucker's essential requirements for an optimal solution have been met.

(c) If this is not the case,

(i) Select $q = q_1$ or q_2 based on $|\lambda_{q_1}| = \max(|\lambda_{q_1}|, |\lambda_{q_2}|)$;

(ii) Insert a_q as the new column$S$;

(iii) Insert λ_1 as a new h element;

(iv) Sum up a new relevant column to R.

(d) Multiply S by 1.

(Note: MINOS also has a DOUBLE PRICE alternative, which provides several non-basic variable to be a super base).

Step 3. (Determine the search direction, $p = Zp_s$).

(a) Complete $R^T R p_S = -h$.

(b) Complete LU $p_B = -Sp_S$.

(c) Make $p = \begin{bmatrix} p_B \\ p_S \\ 0 \end{bmatrix}$.

Step 4. (Test Ratio, "CHUZR").

(a) If $\alpha_{\max} \geq 0$, the highest α value of $x + \alpha p$ is feasible.

(b) If $\alpha_{\max} = 0$, proceed to step 7.

Step 5. (Line search).

(a) Determine α, an α^* approximation in which

$$F(x + \alpha^* p) = \min_{0<\theta\leq\alpha_{\max}} f(x + \theta p)$$

(b) Convert x to $x + \alpha p$ and f and g to their respective values in the new x.

Step 6. (Calculate the reduced slope, $\bar{h} = Z^T g$).

(a) Complete $U^T L^T \pi = g_B$.

(b) Determine the new reduced slope, $\bar{h} = g_S - S^T\pi$.

(c) Using α, p_S and metric-variable recursion on $R^T R$, modify R and switch in reduced gradient, $\bar{h} - h$.

(d) Set $\bar{h} - h$.

(e) If $\alpha < \alpha_{\max}$ proceeds to step 1. Because no new constraints are discovered, they remain in this subspace.

Step 7. (Exchange base if required; eliminate one superbasis). Here, $\alpha < \alpha_{\max}$ has reached one of its limits, and for some $p(0 < p \leq m + s)$, the variable associated to the p column of $[B \quad S]$ has also attained one of its limits.

(a) If the base variable exceeds the limit $(0 < p \leq m)$,

(i) Replace the p-th column with the q-th column of

$$\begin{bmatrix} B \\ X_B^T \end{bmatrix} \text{ and } \begin{bmatrix} S \\ X_S^T \end{bmatrix}$$

where q is picked to maintain B nonsingular (this involves πp vector that fulfills $U^T L^T \pi_p = e_p$);

(ii) Changes to L, U, R and π as well as changes to B to reflect these changes;

(iii) Find the latest lower gradient $h = g_S - S^T\pi$;

(iv) Go to (c).

(b) If not, the variable superbase reaches its limit $(m < p \leq m + s)$. Determine $q = p - m$.

(c) At the appropriate limit, construct the qth variable in nonbasis$S$, as follows:

(i) Remove the qth column from

$$\begin{bmatrix} S \\ X_S^T \end{bmatrix} \text{ and } \begin{bmatrix} R \\ h^T \end{bmatrix};$$

(ii) Add R to the triangular matrix.

(d) Subtract s by one and return to step 1.

5. CONCLUSIONS

Portfolio optimization problems are typically represented by a non-linear program. The objective function contains nonlinearities, which are always quadratic in form. A mathematical method, namely the active constraint, is required to solve this portfolio optimization problem. The steps for completing this portfolio optimization model begin with the Langrange function and then proceed with the algorithm. As a result, the portfolio optimization problem that has been developed and can be solved by deciding which investment to take with the decision variable (binary) is *w*.

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