CODING GAIN ANALYSIS OF ORTHOGONAL POLYNOMIAL BASED SPACE-TIME BLOCK CODE

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ABSTRACT

In recent years, wireless communication is increasingly used in voice and messaging due to the mobility it offers. There is a growing demand for high data rate wireless communication because of the possibility of using this for internet and multimedia applications in the near future. To achieve high data rate communication, the system has to overcome problems such as multipath fading and cochannel interference. The channel statistic is significantly often Rayleigh which makes it difficult for the receiver to reliably determine the transmitted signal unless some less attenuated replica of the signal is provided to the receiver. This technique is called diversity, which can be provided using temporal, frequency, polarization and spatial resources. However, in many situations, the wireless channel is neither significantly time variant nor highly frequency selective. This forces the communication engineers to consider the possibility of deploying multiple antennas at both the transmitter and receiver to achieve spatial diversity and also provide high performance. The space-Time Block code is a technique that can provide diversity and coding gain. In this paper we propose coding gain analysis of the orthogonal polynomials based on Space-Time Block Code.

SECTION I.
INTRODUCTION

1.1 Literature Review

The Space-Time Codes first used in Tarokh et.al. [1], achieve coding gain,with inputs mapped on the vectors rather than scalars. Teletar et.al.[2] and Foshini et.al.[3] had shown independently that the rich scattering wireless channel can support higher data rates when multiple antennas are used at the transmitter and the receiver. Alamouti [4] gave a simple, single symbol decoding STBC for two transmitter antennas from the complex orthogonal design matrix. The simple design rule and an easy decoding technique stimulated a lot of researchers to work in this area and since then many more STBCs, are proposed. In [5], Tarokh et.al. generalized the Alamouti scheme and gave STBC from Orthogonal design. Then many more complex Orthogonal design matrices with rates less than 1 like those in Weifeng Su et.al..[6] were identified. In [7], Sethuraman et.al. proposed STBC from diversion Algebra and extension field concepts to identify code matrices. In [8], Damen et al. proposed a diagonal algebraic space-time code that by construction are of full rank and of rate 1. But these codes require some parameters to optimize over for diversity and coding gain. In our earlier work, [9] we have proposed a new STBC based on Orthogonal polynomials. In [10], Mukkavilli et.al.used the equal eigenvalue criterion to maximize the coding gain and identified some codes using this strategy. In this paper, the data rate improvement and rank analysis of Orthogonal polynomials Space-Time Block code(OPSTBC) are presented.

1.2 System Model

Let us consider, the single-user wireless communication links consisting of Nt transmit antennas and Nr receive antennas. The received symbol rjk can be given as,

$$r_{jk} = \sum_{i=1}^{N_t} h_{ji} u_{ik} + n_{jk}$$

where j = 1,2,..,Nr denote the receive antenna and k = 1,2,…T, the time at which the symbol was sent, uik is the code symbol transmitted from antenna i = 1,2,…,Nt at time k and hji is the complex channel gain between the ith transmit antenna and the jth receive antenna. The noise symbols njk are white Gaussian with mean zero and variance σ2. In matrix formulation, this system can be represented as,

R = U.H + N

where,

H - the channel matrix of dimension Nt x Nr

U - the code matrix of size T x Nt

R - the received matrix of size T x Nr

N - the noise matrix of size T x Nr

Here U is the space-time block code matrix. The space-time block code (STBC) spans a matrix of size Nt x T, where the ith row vector is transmitted by the ith transmit antenna and the tth column vector is transmitted during the tth time slot. We assume quasi-static fading channels where the channel matrix remains constant over the code duration T. Perfect channel estimation at the receiver end is also assumed and the systems have no feedback. Channel estimation is done with training/pilot sequences in regular intervals during the transmission. We focus on full diversity designs that have a simple and effective decoding strategy.

SECTION II

2.1. STBC Based On Orthogonal Polynomials

The essential construction principle of the proposed Orthogonal Polynomial Based Space Time Block Code(OPSTBC) [9] is briefed hereunder:

The proposed space time block coding is considered around a cartezian coordinate separable, blurring, "code operator" in which the signal I results in the super position of point source of impulse weighted by the value of the object function f. Expressing the object function f in terms of derivatives of the signal function I relative to the cartezian coordinates and time is very useful for analyzing the signal in order to achieve the diversity. Hence, the initial requirements to analyse the diversity may be stated as follows: Since the diversity can be achieved based on the local properties of the signal, a local code operator is required to be devised such that it is cartezian separable and denoising operator. The two dimensional code function M(x,y) can be considered to be a real valued function for (x,y) ∈ X×Y where X and Y are ordered subsets of real values. In our case the x is modeled to represent the space and y represents time slot, and consisting of a finite set, which for convenience can be labeled as {0,1,2…,n-1}, the functions M(x,y) reduces to a sequence of functions

$$M(i,t) = u_i(t), \quad i=0,1,2,\dots n-1 \qquad \dots(1)$$

As shown in equation 2 the process of space – time block codes analysis can be viewed as the linear two dimensional transform coding defined by code operator, M(x,y) M(i,t) = ui(t)

$$\beta'(\zeta,\eta) = \int_{x\varepsilon X}\int_{y\,\varepsilon\, Y} M(\zeta, x)\, M(\eta,y)\, I(x,y)\,dx\,dy \qquad \dots(2)$$

Considering both X and Y to be finite set of values {0,1,2…,n-1} equation 1 can be written in matrix notation as follows

$$\left|\beta'_{ij}\right| = (\left|\mathrm{M}\right| \otimes \left|\mathrm{M}\right|)^t \left|\mathrm{I}\right|$$

where the code operator |M| is

$$|M| = \begin{vmatrix} u_o(t_o) & u_1(t_o) & \dots & u_{n-1}(t_o) \\ u_o(t_1) & u_1(t_1) & \dots & u_{n-1}(t_1) \\ & \vdots & & \\ u_0(t_{n-1}) & u_t(t_{n-1}) & \dots & u_{n-1}(t_{n-1}) \end{vmatrix} \qquad \dots(3)$$

⊗ is the outer product $\left|\beta'_{ij}\right|$ and $\left|\mathrm{I}\right|$ are the n2 matrices arranged in the dictionary sequence. $\left|\mathrm{I}\right|$ is the signal to be transmitted and $\left|\beta'_{ij}\right|$ are the coefficients of transformation.

We consider a set of orthogonal polynomials u0(t), u1(t),…,un-1(t) of degrees 0,1,2,…, n-1, respectively. The generating formula for the polynomials is as follows.

$$u_{i+1}(t) = (t-\mu)\, u_i(t) - b_i(n)\, u_{i-1}(t) \text{ for } i \geq 1, \qquad \dots(4)$$

$u_1(t) = t - \mu$, and $u_0(t) = 1$,

where

$$b_i(n) = \frac{<u_i, u_i>}{<u_{i-1}, u_{i-1}>} = \frac{\sum_{t=1}^{n} u_i^2(t)}{\sum_{t=1}^{n} u_{i-1}^2(t)}$$

and

$$\mu = \frac{1}{n}\sum_{t=1}^{n} t$$

Considering the range of values of t to be t =i , i = 1,2,3,…,n, we get

$$b_i(n) = \frac{i^2(n^2 - i^2)}{4(4i^2 - 1)}, \ \mu = \frac{1}{n}\sum_{t=1}^{n} t = \frac{n+1}{2}$$

We can construct code operators $|M|s$ of different sizes from the above orthogonal polynomials. The code operator in equation 3 that defines linear transformation of signal can be obtained as $|\mathrm{M}| \otimes |\mathrm{M}|$, where $|\mathrm{M}|$ is computed and scaled from equation 4 as

$$|M| = \begin{vmatrix} u_0(t_0) & u_1(t_0) & u_2(t_0) \\ u_0(t_1) & u_1(t_1) & u_2(t_1) \\ u_0(t_2) & u_1(t_2) & u_2(t_2) \end{vmatrix} = \begin{vmatrix} 1 & -1 & \frac{1}{3} \\ 1 & 0 & -\frac{2}{3} \\ 1 & 1 & \frac{1}{3} \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & -2 \\ 1 & 1 & 1 \end{vmatrix}$$

…(5)

2.2 Analysis Of The Code

2.2.1. Construction of code matrix U3

In this subsection we present the construction of code matrix of (OPSTBC). Let S = {u1, u2, u3} be the symbols to be transmitted in the three symbol duration. The code matrix U3 is obtained from |M| using the steps given below.

1. The matrix has 3 symbols which are transmitted in the 3 symbol periods.

2. The three symbols are placed in the first row one at each entry and the subsequent rows are filled by the left cyclic rotations of the previous row, i.e.

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ u_2 & u_3 & u_1 \\ u_3 & u_1 & u_2 \end{bmatrix}$$

3. The signs from the matrix |M| are placed adjacent to the corresponding entry of the matrix U3 i.e.

$$\begin{bmatrix} u_1 & -u_2 & u_3 \\ u_2 & u_3 & -u_1 \\ u_3 & u_1 & u_2 \end{bmatrix}$$

4. The entries with a 2 in U3 is added with the next entry in the right rotation of the symbol set S, i.e. making

$$U3 = \begin{bmatrix} u_1 & -u_2 & u_3 \\ u_2 & u_3(0) & -(u_1 + u_2) \\ u_3 & u_1 & u_2 \end{bmatrix}$$

2.2.2 DIVERSITY ORDER OF THE CONSTRUCTED CODE

The code difference matrix ΔU3 has the same structure as the code matrix U3 and an explicit evaluation of the determinant of ΔU3 gives,

$$|\Delta U_3| = u_1'^3 + u_2'^3 + u_1'^2 u_2' + u_2'^2 u_3 + 2u_1' u_2' u_3'$$

….(6)

The entries u1', u2' and u3' in equation(6) take values from the symbol difference set instead of the original symbol set. This equation will become zero, only when the 3-tuple of difference entries becomes zero, i.e. (u1', u2', u3') = (0, 0, 0). This case occurs when the code matrix U and erroneous matrix e received by the receiver become one and the same. As the determinant is non-zero, the code difference matrix is of full rank.

The Peak to Average Power (PAPR) associated with this code matrix U3 is low due to the fact that the number of symbol additions in this matrix is minimum. The code matrix U3 can be rewritten as,

$$U_3 = u_1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} + u_2\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} + u_3\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

….(7)

= u1E1+u2E2+u3E3

where,

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} E_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} E_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

are the basis matrices. Here the code matrix is expressed as a linear combination of basis matrices E1, E2 and E3 weighted by the transmitted symbols u1, u2, u3. Alternatively, we represent code matrix U3 with the basis matrices by the following representation.

U3 = $\overline{U} \bullet \overline{\overline{E}}^T$

where,

$\overline{u}$ = [u1 u2 u3

$\overline{\overline{E}}$ = [E1 E2 E3

and the operator "•" is defined for the product of a vector and a matrix. We detail, the analysis below and derive codes for a better coding gain through the basis matrices.

2.3 Coding Gain Improvement

Coding gain of a diversity 'r' system is defined as the minimal determinant value over all possible code difference matrices. We analyze the coding gain of the proposed STBC with some of well-known symbol sets like the QPSK, BPSK.

The basis matrices for the code matrix U3 (equation 7), can be alternatively seen as vectors in a 9 dimensional space. They are represented as,

E1’’ = [1 0 0 0 0 -1 0 1 0 T

E2’’ = [0 -1 0 1 0 -1 0 0 1 T ….(8)

E3’’ = [0 0 1 0 0 0 1 0 0 T

These vectors are a 3 dimensional subspace to a 9 dimensional space. Here the -1s and 1s correspond to translate on either side of the origin. Upon studying the coding gain and hence the performance of the code with these subspaces, it is found that there are redundant information in some dimension that can be avoided by pruning the entries in those co-ordinates and shifting them back to the origin. In the proposed coding only one redundant information is present. By pruning that redundant information, we maximize the average energy per symbol in the transmission and minimize the PAPR value as number of symbol additions get reduced. We use the notation U31 for the new matrix obtained from U3 and G for the corresponding basis matrices.

$$U_{31} = \begin{bmatrix} u_1 & -u_2 & u_3 \\ u_2 & u_3(0) & -u_1 \\ u_3 & u_1 & u_2 \end{bmatrix} \quad ...(9)$$

where

$$G_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} G_{12} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} G_{13} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

….(10)

This matrix has one co-ordinate in the basis set of G12 shifted towards zero. For the QPSK and BPSK symbol set, this code is verified to be of full rank, thereby justifying the claim. By doing this, we have avoided spending more power in the redundant values corresponding to U3 as the average symbol power gets increased by some value and hence coding gain. The determinant of the corresponding code difference matrix U31 is given by,

$$|\Delta U_{31}| = u_1'^3 + u_2'^3 + 2u_1'u_2'u_3' \quad ….(11)$$

When, we compare the equations (6) and (11), the determinant value of both original and code difference matrices are homogenous and symmetric equations. This is the best that can be obtained from the U3 code as the system loses its full rank nature after this stage.

SECTION III

3.1 Data-Rate Improvement Using the Code U3

When using multiple antennas for transmission/reception, an interesting issue is with the data rate that the system can support. It is about the number of distinct symbols per channel usage that this code can support and another is with the decoding of these codes. We show ways to add upto nine different symbols per code word matrix transmission when using the BPSK, QPSK symbol set with the code matrix U3 and U3M.

Data rate improvements with STBC were earlier tried out with the extension field codes by Sethuraman et al[8]. We use similar approaches here, but restrict ourselves to extensions given by rotations on the unit circle of the form $e^{j\theta}$, where θ is a value to optimize. These extensions retain the diversity nature of the code matrix when packing the extra information into the code.

This code U3M, when coded at the rate of 6 symbols per transmitted code word will be given by,

$$U_{32M} = \begin{bmatrix} y_1 & -y_2 & y_3 \\ y_2 & y_3 & -y_1 \\ y_3 & y_1 & y_2 \end{bmatrix} \quad ….(12)$$

where,

$$y_1 = u_{11} + \gamma . u_{12}$$

$$y_2 = u_{21} + \gamma . u_{22}$$

$$y_3 = u_{31} + \gamma . u_{32}$$

And $\gamma = e^{-i\pi/3}$ for BPSK symbol set.

Similarly to pack 9 symbols per transmitted code word the new code matrix will be given by,

$$U_{33M} = \begin{bmatrix} z_1 & -z_2 & z_3 \\ z_2 & z_3 & -z_1 \\ z_3 & z_1 & z_2 \end{bmatrix} \quad ….(13)$$

where

$$z_1 = u_{11} + \gamma.u_{12} + \gamma^2.u_{13}$$
$$z_2 = u_{21} + \gamma.u_{22} + \gamma^2.u_{23}$$
$$z_3 = u_{31} + \gamma.u_{32} + \gamma^2.u_{33}$$

and

$\gamma = e^{-i\pi/4}$ for the BPSK symbol set.

3.2 Decoding the HIGH RATE CODES

Once coded to achieve a higher data rate, another issue remains as to how to decode the system. We use the sphere decoder in decoding and outline the steps over it. We explain how to decode the system of equation in the first case of 6 symbols per transmitted code word. We define H as the equivalent matrix seen in the system model. The system equation is given by,

r=h.y+n

where, y = [y1 y2 y3 T . This system of equation can be rearranged as,

r = H'.y'+ n

where y' = [u11 u21 u31 u12 u22 u32 T

H' = [H γ .H

As proposed in [17], in this work, we use the reduced complexity sphere decoder,as this sphere decoder makes usage of the information in the channel and gives a performance close to ML.

SECTION IV

SIMULATION

In this section we present the simulation results corresponding to the code U3 and its improved version U3M. In figure 1, the comparison details for code U3M with U3 are given for the QPSK symbol set with two receive antennas. It can be seen that the code U3M performs better than the code U3. The comparisons show around 3dB improvement in performance at a BER of 10-5. Thus, the elimination of redundancy and hence the improvement in the average signal-to-noise ratio has reflected in the system performance

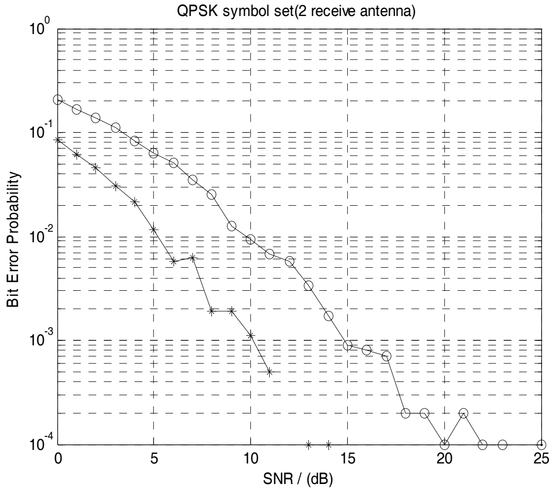

Figure 1: Comparison of the codes U3 and U3M for the QPSK symbol set

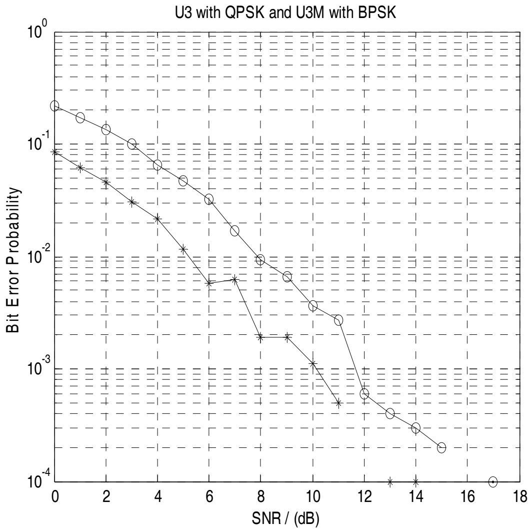

Figure 2: Comparing codes U3 with QPSK and code U3M with BPSK

In figure 2, the performance of the high rate code U32M is compared with a same rate U3 code. The performance comparison was done for the code U3M with QPSK modulation and code U32M with BPSK modulation, to achieve a data-rate when 6 symbols / code matrix is used. The performance plots show more than 3 dB improvement in the gain.

SECTION V

CONCLUSION

In this paper, we construct code matrix U3 which is obtained from M3. We prove that code matrix U3 has full rank nature. We have analysed the coding gain of the OPSTBC with some of well known symbol sets such as BPSK and QPSK. The proposed code matrix U3 has one redundant information. By pruning that redundant information we obtain U3M. By comparing U3 and U3M matrices, the performance results are obtained. Based on the performance results, it is proved that the coding gain of U3M code is relatively higher. Also we utilize Cholskey factorization technique for faster search of a symbol in sphere decoding algorithm by minimizing the boundary level of search.

SECTION VI

REFERENCES:

[1] Vahid Tarokh, Nambi Seshadri and A.R. Calderbank. 'Space Time Codes for High Data Rate Wireless Communication; Performance Criterion and Code Construction', IEEE Transcations of Information Theory, vo. 44, No. 2, pp. 744 – 765. 1998.

[2] Teletar I.E. 'Capacity of multi-antenna Gaussian channels', AT & T Bell labs, Tech. Report and European Trans of Telecommunications, Vol. 10, pp. 585 – 595.1995.

[3] Foschini G.J. and M.J. Gans. 'On the limits of wireless communication in a fading environment when using multiple antennas', Wireless Personal Communication, Vol. 6, pp. 311 – 335. 1998.

[4] Siavash L. Alamouti. 'A Simple Transmit Diversity Technique for Wireless Communication', IEEE Journal of Selected Areas in communication, Vol. 16, No 8, pp. 1451 – 1458. 1998.

[5] Vahid Tarokh, Hunid jafarkhani and A.R. Calderbank. 'Space-Time Block coding for Wireless Communication: Performance Results', IEEE Journal on Selected Areas in Communication, Vol. 17. No. 3, pp. 451 – 460. 1999.

[6] Weifeng Su and Xiang-Gen Xiz. 'Two Generalized Complex Orthogonal Space-Time Block Codes of Rates 7/11 and 3/5 for 5 and 6 Transmit antennas', IEEE Transactions on Information Theory, Vol. 49, No. 1, pp. 313 – 316. 2003.

[7] B.A. Sethuraman, B. Sundar Rajan and V. Shashidhar. 'Full-diversity, High rate Space-Time Block Codes from Division Algebras', IEEE Transactions on Information Theory, Vol. 49, No. 10, pp. 2596 – 2616. 2003.

[8] M.O. Damen, K. Abed-Meraim and J.C. Belfiore. 'Diagonal Algebraic Space-Time Block codes', IEEE Transaction on Information Theory Vol. 48, No.3, pp. 628 – 636. 2002.

[9] R.Krishnamoorthy and K.Selvakumar, 'An Efficient Space Time Block Code based on Orthogonal Polynomials' IEEE Communication letter (Under review)

[10] Krishna Kiran Mukkavilli, Mihai Ionescu and Behnaam Aazhang. 'Design of Space-Time Codes with Optimal Coding Gain', Proc. Of PIMRC, London, UK, pp. 495 499. 2000.