PERFORMANCE ASSESSMENT OF NHPP SOFTWARE RELIABILITY MODELS APPLYING PARETO-TYPE LIFETIME DISTRIBUTION PROPERTIES

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ABSTRACT

This study compares and evaluates the performance of NHPP-based software reliability models that apply Pareto-type lifetime distribution characteristics and proposes the optimal model based on this analysis. To analyze software failure phenomena, failure time data were utilized, and the parameters of the proposed model were determined through the maximum likelihood estimation (MLE) method. Various analyses (including assessing model efficiency using MSE and R^2, evaluating prediction accuracy against true values using the mean value function, measuring failure occurrence intensity using the intensity function, and assessing future reliability using the reliability function) demonstrated that the Lomax model exhibited the best performance. Therefore, this research provides new insights into the reliability performance of Pareto-type lifetime distributions, which have been underexplored in existing studies, and also offers fundamental reliability attribute data that software developers need in the early stages.

Keywords: *Goel-Okumoto, Lomax, NHPP, Pareto, Reliability Performance.*

1. INTRODUCTION

The era of the Fourth Industrial Revolution can be described as an information society based on Artificial Intelligence (AI), where data generated and collected through advanced information and communication infrastructure is combined with AI technologies to create new value in a software-centric age. Therefore, the current industrial paradigm is converging around software, evolving into a digital form in conjunction with related industries. As a result, in this information age, which merges with digital technologies based on advanced AI software capabilities, the reliability of high-quality software free from defects becomes the most critical issue. Therefore, software developers conduct reliability testing to verify whether user requirements are met during the development stage before releasing the software. Furthermore, to proceed with software development more efficiently, it is essential not only to perform reliability testing but also to predict and eliminate software defect occurrence characteristics in advance [1]. Research aimed at predicting defect occurrences using failure times that arise during software system operation is actively underway, and many reliability models related to this have been proposed to date [2]. Especially, Kim [3] demonstrated the efficiency of NHPP-based models under the premise of finite failures using a Rayleigh distribution model applied to Non-homogeneous Poisson Processes (NHPP). Additionally, Min [4] evaluated the characteristics of the NHPP reliability models using Pareto and Erlang distributions. Moreover, Kim [5] analyzed and presented the properties affecting the reliability of software reliability models using Lomax and Minimax lifetime distributions. Furthermore, Yang [6] examined the properties of software development models with infinite failures, utilizing the Lomax distribution, which is a generalized form of the Pareto-type distribution. Similarly, Satya, Sita, and Sridevi [7] provided analysis results of NHPP reliability models incorporating Pareto distribution characteristics. Based on these research findings, recent studies have focused on applying NHPP-based reliability models to software failure times in order to improve the reliability performance [8].

Therefore, this study aims to propose an optimization method for analyzing the performance characteristics of NHPP software reliability models based on the Pareto-type distribution, along with presenting the optimal model.

2. RELATED RESEARCH

2.1.1 NHPP model

The NHPP model is a probability-based prediction model suitable for explaining events whose occurrence rate (occurrence frequency) randomly changes over time or under specific conditions, such as software failures. Therefore, it is widely used in fields such as software reliability and failure analysis.

Based on this, in the method of formulating the NHPP model, N(t) is the accumulated failure count, m(t) is the mean value function, and λ(t) is the intensity function, implying that N(t) follows a Poisson probability distribution, where m(t) serves as the parameter.

Therefore, if this is organized into a formula, it can be defined as Equation (1) [9].

$$P\{N(t)=n\}=\frac{[m(t)]^n\cdot e^{-m(t)}}{n!} \tag{1}$$

Note that $n = 0,1,2,\cdots\infty$.

Furthermore, in terms of reliability performance, m(t) is an attribute function that reflects the expected number of failures, and λ(t) is an attribute function representing the instantaneous failure occurrence rate. Therefore, these attribute functions can be organized into relationships such as Equations (2) and (3).

$$m(t)=\int_0^t \lambda(s)ds \tag{2}$$

$$\frac{dm(t)}{d(t)}=\lambda(t) \tag{3}$$

2.1.2 NHPP software reliability model

The NHPP software reliability model can be said to be a probability model that can statistically explain and predict the phenomenon in which the failure rate of a software system changes over time. It takes into account the changing nature of defect occurrences over time and is particularly useful for analyzing situations where defect occurrence intervals are not uniform. Therefore, software developers can use this model to predict the number of defects, plan the necessary testing time and resources, and assess the expected reliability of the software before its final release.

This study assumes the finite failure condition, which posits that no further defects occur during fault repair. In this case, if the remaining detectable faults by testing time t are θ, with the cumulative distribution function as F(t) and the probability density function as f(t), the attribute function reflecting the model's performance is expressed in Equations (4) and (5).

$$\mathrm{m}(\mathrm{t}|\theta,\mathrm{b})=\theta F(\mathrm{t}) \tag{4}$$

$$\lambda(\mathrm{t}|\theta,\mathrm{b})=\theta F(\mathrm{t})'=\theta f(t) \tag{5}$$

Accordingly, the likelihood function of the NHPP model is as shown in Equation (6).

$$L_{NHPP}(\Theta|\underline{x})=\left(\prod_{i=1}^{n}\lambda(x_i)\right)exp[-m(x_n)] \tag{6}$$

Note that $\underline{x}=(x_1,x_2,x_3\cdots x_n)$

2.2 NHPP Goel-Okumoto Basic Model

The Goel-Okumoto model is useful for evaluating reliability based on defects found in the software testing phase, and is used as an important tool for establishing reliability prediction and quality assurance strategies. Also, this model is also called the Exponential-type basic model because its life distribution has an exponential shape, and is treated as a representative basic model in the reliability testing field.

Therefore, if the residual failure is θ, the attribute function expressing the reliability performance is defined as in Equations (7) and (8) [10].

$$m(t|\theta,b)=\theta(1-e^{-bt}) \tag{7}$$

$$\lambda(t|\theta,b)=\theta be^{-bt} \tag{8}$$

Accordingly, after inserting Equations (7) and (8) into Equation (6), taking the logarithm on both sides, the log-likelihood function is derived as follows.

$$lnL_{NHPP}(\Theta|\underline{x})=nln\theta+nlnb-b\sum_{k=1}^{n}x_k-\theta(1-e^{-bx_n}) \tag{9}$$

By differentiating with respect to the model parameters θ and b, respectively, we can calculate the maximum likelihood estimators $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ that satisfy Equations (10) and (11). Thus, by applying the bisection method to solve the following equation, the model parameters can ultimately be determined.

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial\theta}=\frac{n}{\hat{\theta}}-1+e^{-\hat{b}x_n}=0 \quad (10)$$

$$\frac{\partial lnL_{NHPP}(\Theta|\underline{x})}{\partial b}=\frac{n}{\hat{b}}-\sum_{i=1}^{n}x_n-\hat{\theta}x_ne^{-\hat{b}x_n}=0 \quad (11)$$

2.3 NHPP Lomax Model

The Lomax distribution is a special case of the Pareto-type II distribution, and is mainly used in reliability lifetime distribution analysis. In other words, it is a modified form of the Pareto basic distribution, and is mainly useful for analyzing situations where the frequency of significant events or extreme values appear.

Therefore, it has characteristics suitable for data where extreme values frequently occur, and is a distribution widely used for implementing optimization models in various fields, including the reliability field.

Therefore, the probability function of the Lomax distribution is as shown in Equations (12) and (13).

$$\mathrm{F(t)}=1-(1+bt)^{-a} \quad (12)$$

$$\mathrm{f(x)}=\frac{ab}{(1+bt)^{a+1}} \quad (13)$$

Note that b (> 0) is the scale parameter.

That is, the attribute function expressing the reliability performance can be derived as in Equations (14) and (15) [11].

$$\mathrm{m(t}|\theta,\mathrm{b})=\theta\left[1-(1+bt)^{-a}\right] \quad (14)$$

$$\lambda(\mathrm{t}|\theta,\mathrm{b})=\theta\left[\frac{ab}{(1+bt)^{a+1}}\right] \quad (15)$$

By inserting Equations (14) and (15) into Equation (6), setting the shape parameter a to 1, and taking the logarithm of both sides, the log-likelihood function is derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x})=nln\theta+nlnb \quad (16)$$

$$-2\sum_{i=1}^{n}\ln(1+bx_i)-\theta\left[1-(1+bx_n)^{-1}\right]=0$$

By differentiating with respect to the model parameters θ and b, respectively, we can calculate the maximum likelihood estimators $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ that satisfy Equations (17) and (18). Accordingly, if applying the bisection method to solve the following equation, the model parameters can ultimately be calculated.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta}=\frac{n}{\hat{\theta}}-\left[1-(1+bx_n)^{-1}\right]=0 \quad (17)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b}=\frac{n}{\hat{b}}-2\sum_{i=1}^{n}\frac{x_i}{(1+\hat{b}x_i)}-\hat{\theta}x_n\left[\frac{1}{\left(1+\hat{b}x_n\right)^2}\right]=0 \quad (18)$$

2.4 NHPP Pareto Basic Model

The Pareto distribution is the Pareto-type I, and is the most basic form among the Pareto-type lifetime distributions. This distribution is suitable for explaining extreme events or phenomena in which a small number of causes lead to a large number of results. Therefore, it is a useful distribution for analyzing unbalanced phenomena because it has mathematical characteristics and characteristics that reflect observable phenomena.

Thus, a reliability analysis model using the Pareto distribution can be an important tool for identifying system failure characteristics and designing effective maintenance strategies. It is particularly useful in systems in which the distribution of failures is unbalanced, and through this, system reliability can be improved and operational efficiency can be improved.

Also, the probability function of the Pareto distribution is as shown in Equations (19) and (20).

$$F(\mathrm{t})=1-\left[1+\left(\frac{t}{b}\right)\right]^{-a} \quad (19)$$

$$f(\mathrm{t})=\frac{a}{b}\left[1+\left(\frac{t}{b}\right)\right]^{-(a+1)} \quad (20)$$

Note that b (> 0) is the scale parameter.

That is, the attribute function representing the reliability performance is derived as in Equations (21) and (22) [12].

$$\mathrm{m(t}|\theta,\mathrm{b})=\theta\left(1-\left[1+\left(\frac{t}{b}\right)\right]^{-a}\right) \quad (21)$$

$$\lambda(\mathrm{t}|\theta,\mathrm{b})=\theta\left(\frac{a}{b}\left[1+\left(\frac{t}{b}\right)\right]^{-(a+1)}\right) \quad (22)$$

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Accordingly, the log-likelihood function is formulated by inserting Equations (21) and (22) into Equation (6), setting the shape parameter a to 1, and computing the logarithm on both sides.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta + nlna - nlnb \quad (23)$$

$$+\sum_{i=1}^{n} \ln\left[1+\left(\frac{x_i}{b}\right)\right]^{-(a+1)} - \theta\left(1-\left[1+\left(\frac{x_n}{b}\right)\right]^{-a}\right)$$

Accordingly, if differentiating Equation (23) by each parameter (θ and b) and solving it using the bisection method, we can obtain the maximum likelihood estimators ($\hat{\theta}_{MLE}$, $\hat{b}_{MLE}$) that satisfy Equations (24) and (25).

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - \left(1-\left[1+\left(\frac{x_n}{b}\right)\right]^{-a}\right) = 0 \quad (24)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = -(\hat{b}-1)\frac{x_n\hat{\theta}a}{\hat{b}^2}\left[1+\left(\frac{x_n}{\hat{b}}\right)\right]^{-(a+1)}$$

$$-\frac{n}{\hat{b}} + (a+1)\left(\frac{n}{\hat{b}} - \sum_{i=1}^{n}\frac{1}{\hat{b}+x_i}\right) = 0 \quad (25)$$

3. RELIABILITY PERFORMANCE ANALYSIS OF THE PROPOSED MODEL

This study focuses on assessing the reliability performance by analyzing the software failure time observed during the operation of a desktop computer system.

The failure time is based on data collected from a total of 30 failures that occurred randomly over a total of 738.68 hours, as shown in Table 1 [13]. Also, the failure time is presumed to be caused by testing mistakes and design errors made by developers during the initial stage of software development.

To assess whether the failure time data collected in this study is suitable for reliability analysis, a simulation was carried out using the Laplace Trend Test with the data presented in Table 1, and the results are shown in Figure 1. Typically, if the Laplace trend test result for the referenced data falls within the range of '-2 to 2', it indicates that there are no extreme values, which means that the data are stable and therefore suitable for reliability analysis.

Table 1: Software Failure Time.

Failure number	Failure time (hours)	Failure time (hours)$\times 10^{-2}$
1	30.02	0.30
2	31.46	0.31
3	53.93	0.53
4	55.29	0.55
5	58.72	0.58
6	71.92	0.71
7	77.07	0.77
8	80.90	0.80
9	101.90	1.01
10	114.87	1.14
11	115.34	1.15
12	121.57	1.21
13	124.97	1.24
14	134.07	1.34
15	136.25	1.36
16	151.78	1.51
17	177.50	1.77
18	180.29	1.80
19	182.21	1.82
20	186.34	1.86
21	256.81	2.56
22	273.88	2.73
23	277.87	2.77
24	453.93	4.53
25	535.00	5.35
26	537.27	5.37
27	552.90	5.52
28	673.68	6.73
29	704.49	7.04
30	738.68	7.38

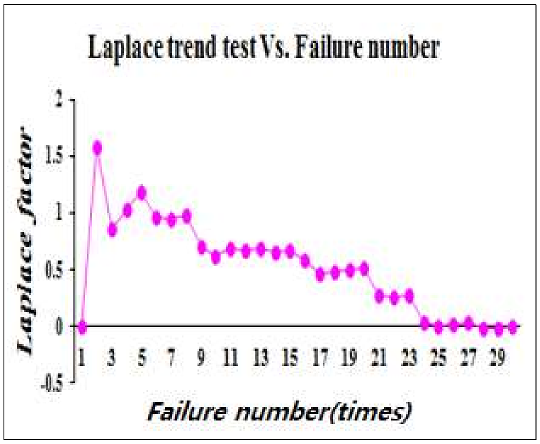

Figure 1: Analysis Results of Laplace Trend Test.

Figure 1 illustrates that the trend test analysis results reveal no extreme values, as all values lie between -2 and 2. Consequently, the failure time data shown in Table 1 is considered appropriate for this work [14].

Also, the NHPP model parameters ($\hat{\theta}$, $\hat{b}$) were estimated via the MLE method, and the findings are summarized in Table 2.

Table 2: Parameter Solution using MLE.

Type	NHPP model	MLE	
		$\hat{\theta}$	$\hat{b}$
Basic	Goel-Okumoto	33.4092	0.3090
Pareto-type lifetime distribution	Lomax	44.1466	0.2848
	Pareto	31.8150	0.4468

For reference, among the parameters presented in Table 2, $\hat{\theta}$ represents the residual failure of the software, and $\hat{b}$ represents the shape parameter that creates the form of the lifetime distribution.

3.1. Selection of an Efficient Model

The efficiency of the proposed model was confirmed through the analysis of the R^2 property, used for selecting a suitable and efficient model based on prediction, and the MSE property, which helps identify an accurate and efficient model.

R^2 is an indicator of the explanatory power of the model, called the coefficient of determination, and has a value between 0 and 1. This value shows how well the model explains the variability of the data, so it is used as a reference value that explains the error between the actual value and the observed value. Therefore, R^2 is defined as in Equation (26).

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n} \left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (26)$$

Note that $\hat{m}(x_i)$ is the cumulative number of failures estimated from the m(t) up to time x_i.

Therefore, when selecting an efficient model among the proposed models, the larger the R^2 value, the smaller the error, so the model is considered a relatively useful model.

MSE is calculated by squaring the average error between the model's actual observed values and predicted values, making it an important metric for evaluating model efficiency. Therefore, MSE can be defined as shown in Equation (27).

$$MSE = \frac{\sum_{i=1}^{n} \left(m(x_i) - \hat{m}(x_i)\right)^2}{n - k} \quad (27)$$

Note that n is the number of observed failures.

Figure 2 shows the results of analyzing the performance of the proposed model using MSE. In this work, the results of MSE analysis will be used as reference data to determine an efficient model.

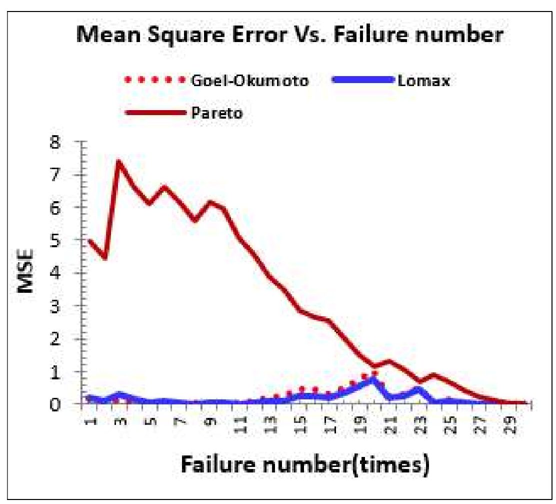

Figure 2: Property Analysis of MSE.

Therefore, when selecting an efficient model among the proposed models, the smaller the MSE, the better the model's predictive performance can be evaluated. That is, the MSE value represents how accurately the predicted values match the actual values, with a lower MSE corresponding to a smaller prediction error. Hence, a lower MSE value indicates superior model performance.

Table 3 shows the data that compares and analyzes in detail the changes in MSE values according to each failure in order to select an efficient model among the proposed models [15].

Table 3: Transition Analysis Data Using MSE.

Failure number	MSE		
	Goel-Okumoto	Lomax	Pareto
1	0.1371	0.2191	4.9607
2	0.0428	0.0949	4.4365
3	0.1617	0.2957	7.4053
4	0.0555	0.1436	6.6016
5	0.0105	0.0627	6.0985
6	0.0154	0.0809	6.6288
7	0.0002	0.0319	6.1659
8	0.0133	0.0025	5.5764
9	0.0000	0.0308	6.1450
10	0.0000	0.0278	5.9483
11	0.0345	0.0002	5.0845
12	0.0844	0.0149	4.5318
13	0.1885	0.0711	3.8896
14	0.2542	0.1158	3.4739
15	0.4425	0.2523	2.8661
16	0.4356	0.2557	2.6288
17	0.2994	0.1691	2.5302
18	0.4968	0.3262	2.0070
19	0.7612	0.5481	1.5321
20	1.0319	0.7870	1.1448
21	0.2603	0.1974	1.3289
22	0.3052	0.2516	1.0232
23	0.5016	0.4368	0.6939
24	0.0507	0.0284	0.8800
25	0.1447	0.0976	0.6797
26	0.0399	0.0174	0.4061
27	0.0045	0.0000	0.2119
28	0.0551	0.0372	0.1204
29	0.0137	0.0076	0.0300
30	0.0000	0.0002	0.0000

In general, if the value of R^2 is greater than 0.8 (80%) and the value of MSE is smaller, it is considered efficient. Therefore, based on the data analyzed in Table 4, it can be judged that the Goel-Okumoto and Lomax models are efficient among the proposed models. In addition, the Lomax model among the models can be evaluated more efficient and useful than the Goel-Okumoto model.

Table 4: Selection Criteria for Efficient Model.

Type	NHPP model	R^2	MSE
Basic	Goel-Okumoto	0.9814	5.8424
Pareto-type lifetime distribution	Lomax	0.9854	4.6059
	Pareto	0.6988	95.031

3.2. Analysis of the Mean Value Function (m(t))

The NHPP reliability model utilizes m(t) as a key function to describe the expected number of software failures by time t. As a result, m(t) represents not only the expected failure rate but also serves as a key parameter in estimating the true value.

Table 5 summarizes and compares the equations for calculating m(t) of the proposed model.

Table 5: Mean Value Function ($m(t)$).

Type	NHPP model	m(t)
Basic	Goel-Okumoto	$\theta(1-e^{-bt})$
Pareto-type lifetime distribution	Lomax	$\theta[1-(1+bt)^{-a}]$
	Pareto	$\theta\left(1-[1+\left(\frac{t}{b}\right)]^{-a}\right)$

Figure 3 shows the trend of the m(t) function, which indicates the ability to predict the true value [16]. Therefore, when analyzing the simulation results, the Pareto model shows a pattern of not accurately predicting the true value, but the Lomax and Goel-Okumoto models show results of roughly predicting the true value.

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Therefore, it was confirmed that these models had efficient performance [16]. In particular, it can be judged that the Lomax model, which showed the smallest error in predicting the true value, is the best.

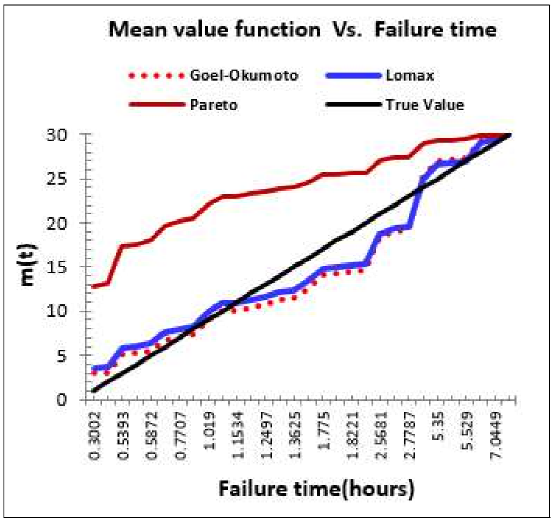

Figure 3: Property Analysis of $m(t)$.

3.3. Analysis of the Intensity Function (λ(t))

In the NHPP reliability model, λ(t) represents the conditional failure rate of software at a given time. Hence, λ(t) indicates the instantaneous failure rate at time t. Consequently, along with m(t), λ(t) functions as a key performance metric for assessing reliability properties.

Table 6 summarizes and compares the equations for calculating λ(t) of the proposed model [17].

Table 6: Intensity Function ($\lambda(t)$).

Type	NHPP model	λ(t)
Basic	Goel-Okumoto	$\theta b e^{-bt}$
Pareto-type lifetime distribution	Lomax	$\theta\left[\frac{ab}{(1+bt)^{a+1}}\right]$
	Pareto	$\theta\left(\frac{a}{b}\left[1+\left(\frac{t}{b}\right)\right]^{-(a+1)}\right)$

Figure 4 shows the results of analyzing the performance of the proposed model with the λ(t) function, which represents the occurrence intensity for instantaneous failures over the entire failure time.

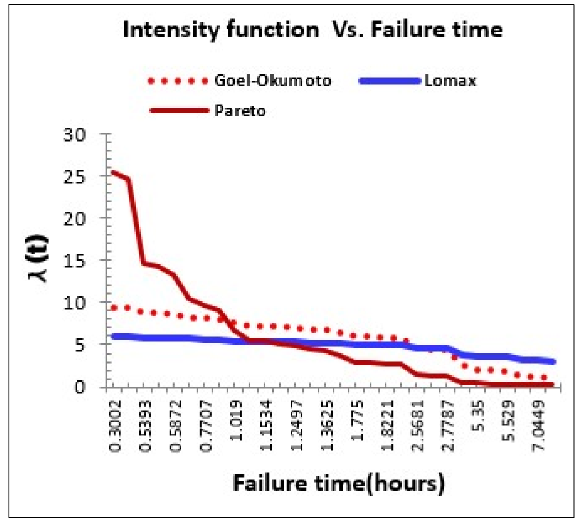

Figure 4: Property Analysis of $\lambda(t)$.

According to the simulation, the Lomax and Goel-Okumoto models initially display a low failure rate, which gradually decreases over time as failures are repaired. This finding indicates that the proposed models achieve efficient performance in terms of model fitness. But, the intensity function of the Pareto model shows the largest failure rate in the initial stage compared to other models, which shows that the model is inefficient in terms of model fitness.

Therefore, this work aims to analyze the occurrence of system failures over time specifically using the results of the intensity function and to help developers establish design and maintenance strategies in the early stage of software development.

So far, based on the analyzed data, it can be confirmed that the m(t) function can predict the occurrence of a failure, and λ(t) can evaluate the possibility of occurrence of a failure. Thus, if the two functions are utilized appropriately, the reliability of the proposed model can be effectively analyzed and the software system can be optimized.

Table 7 shows the data analyzed by comparing the result values in detail according to the failure occurrence time using the attribute function (m(t), λ(t)) that has the greatest influence on the reliability performance. Therefore, in this work, the data presented in Table 7 will be applied to determine the optimal model.

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Table 7: Transition Analysis Data of Reliability Performance Attributes.

Failure Time (hours) $\times 10^{-2}$	Reliability Performance Attributes					
	$m(t)$			$\lambda(t)$		
	Goel-Okumoto	Lomax	Pareto	Goel-Okumoto	Lomax	Pareto
0.3002	2.95970	3.47711	12.7856	9.40889	6.02875	25.4744
0.3146	3.09488	3.63019	13.1455	9.36712	6.01692	24.5199
0.5393	5.12827	5.87780	17.3996	8.73881	5.83812	14.6185
0.5529	5.24687	6.00586	17.5957	8.70216	5.82764	14.2235
0.5872	5.54377	6.32506	18.0674	8.61041	5.80138	13.2955
0.7192	6.65748	7.50519	19.623	8.26628	5.70246	10.4556
0.7707	7.07983	7.94588	20.1394	8.13578	5.66477	9.58975
0.809	7.38959	8.26681	20.4955	8.04006	5.63707	9.01372
1.019	9.02439	9.93003	22.1172	7.53491	5.48986	6.61600
1.1487	9.98234	10.8823	22.9056	7.23890	5.40272	5.58408
1.1534	10.0163	10.9158	22.9317	7.22839	5.39962	5.55132
1.2157	10.4623	11.3538	23.2646	7.09057	5.35878	5.14306
1.2497	10.7021	11.5880	23.4360	7.01647	5.33676	4.93898
1.3407	11.3317	12.1987	23.8625	6.82192	5.27869	4.44890
1.3625	11.4800	12.3416	23.9584	6.77612	5.26496	4.34234
1.5178	12.5074	13.3237	24.5794	6.45863	5.16922	3.68296
1.775	14.1043	14.8234	25.4170	5.96520	5.01810	2.87962
1.8029	14.2700	14.9773	25.4964	5.91399	5.00223	2.80864
1.8221	14.3832	15.0824	25.5498	5.87901	4.99137	2.76131
1.8634	14.6245	15.3057	25.6618	5.80446	4.96817	2.66346
2.5681	18.3002	18.6488	27.1001	4.66867	4.60312	1.56387
2.7388	19.0765	19.3452	27.3527	4.42880	4.52262	1.40076
2.7787	19.2521	19.5026	27.4079	4.37453	4.50421	1.36632
4.5393	25.1923	24.8920	28.9640	2.53901	3.81832	0.57177
5.35	27.0131	26.6536	29.3627	1.97638	3.56813	0.42303
5.3727	27.0578	26.6983	29.3723	1.96257	3.56159	0.41973
5.529	27.3573	27.0000	29.4362	1.87003	3.51724	0.39806
6.7368	29.2423	29.0208	29.8362	1.28756	3.20849	0.27546
7.0449	29.6207	29.4623	29.9175	1.17063	3.13822	0.25327
7.3868	30.0005	29.9230	30.0003	1.05327	3.06376	0.23164

3.4. Analysis of the Reliability Function ($\hat{R}(\tau)$)

$\hat{R}(\tau)$ is a crucial attribute used to assess the reliability performance of software systems with failure rates that vary over time. Together with the previously analyzed functions (m(t), λ(t)), it provides essential data for forecasting the model's reliability. Thus, this study seeks to forecast future reliability performance by utilizing arbitrary mission times in the proposed model.

The function $\hat{R}(\tau)$ represents the likelihood that a failure happens at the testing point t but does not occur within the confidence interval throughout the mission time τ. This means that the reliability function is intended to assess future reliability trends by introducing a randomly assigned mission time following the final failure time ($x_n = 738.68 \times 10^{-2}$). Accordingly, reliability ($\hat{R}(\tau)$) can be defined as shown in Equation (28) [18].

$$\hat{R}(\tau|x_n) = exp[-\{m(x_n + \tau) - m(x_n)\}] \qquad (28)$$

As Figure 5 demonstrates, the analysis of reliability trends over mission time (145H) shows a continuous decline in reliability for the proposed models. That is, all of the proposed models showed a property of decreasing stability as the mission time passed, but among them, the Pareto model, which showed relatively high reliability, can be evaluated as relatively efficient.

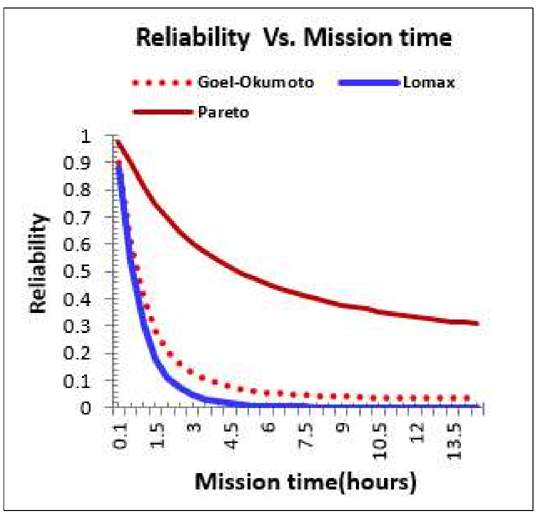

Figure 5: Property Analysis of $\hat{R}(\tau)$.

Table 8 presents a detailed dataset on reliability performance trends derived by applying future mission times to the NHPP model proposed in this study. In reliability performance analysis, $\hat{R}(\tau)$ takes a value of "1 or less," where higher values indicate greater model reliability. Consequently, the model is ultimately considered efficient.

Table 8: Transition Analysis Data of $\hat{R}(\tau)$.

Mission Time (hours)	$\hat{R}(\tau)$		
	Goel-Okumoto	Lomax	Pareto
0.1	0.90148	0.87868	0.97738
0.5	0.61387	0.53582	0.89684
1	0.40410	0.30256	0.81430
1.5	0.28242	0.17891	0.74704
2	0.20777	0.11019	0.6913
2.5	0.15972	0.07036	0.64467
3	0.12749	0.04640	0.60501
3.5	0.10510	0.03150	0.57099
4	0.08907	0.02195	0.54151
4.5	0.07730	0.01565	0.51578
5	0.06840	0.01140	0.4931
5.5	0.06169	0.00847	0.47306
6	0.05642	0.00640	0.45518
6.5	0.05227	0.00492	0.43916
7	0.04896	0.00383	0.4247
7.5	0.04629	0.00303	0.41165
8	0.04410	0.00242	0.3997
8.5	0.04233	0.00196	0.38894
9	0.04086	0.00160	0.37901
9.5	0.03965	0.00132	0.36989
10	0.03863	0.00110	0.36148
10.5	0.03778	0.00093	0.35371
11	0.03707	0.00078	0.34650
11.5	0.03647	0.00067	0.33981
12	0.03597	0.00057	0.33357
12.5	0.03550	0.00050	0.32774
13	0.03518	0.00043	0.32229
13.5	0.03487	0.00038	0.31717
14	0.03461	0.00033	0.31230
14.5	0.03438	0.00029	0.30785

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3.5. Evaluation of Reliability Performance

In this work, we included reference indices (MSE, R^2) that can determine a relatively efficient model in terms of accuracy and fitness of the reliability model, and also used attribute functions (m(t), λ(t), $\hat{R}(\tau)$) that can evaluate performance in terms of reliability. Accordingly, we evaluated the performance using the analyzed results and presented the optimal model [19].

Table 9 presents the outcomes of a thorough comparison and assessment of the attribute data concerning the performance, based on the study findings. The evaluation results confirm that the Lomax model demonstrates the best performance.

Table 9: Evaluation of Reliability Performance.

NHPP model	Model Efficiency		Reliability Performance		
	MSE	R^2	m(t)	λ(t)	$\hat{R}(\tau)$
Goel-Okumoto	Best	Best	Good	Good	Good
Lomax	Best	Best	Best	Good	Good
Pareto	Worst	Worst	Worst	Worst	Best

Therefore, the findings of this study are anticipated to be beneficial in the early stages of software development, providing developers with essential design data and key attributes necessary for enhancing reliability.

4. CONCLUSION

Software developers can design reliability prediction models by utilizing failure time data collected during the early stages of program design and testing. This enables them to predict failure times in advance and improve product reliability based on these predictions. As a result, developers can effectively enhance the software quality they strive to achieve. In this study, we applied an NHPP-based model incorporating Pareto distribution characteristics, which is widely recognized as suitable for reliability analysis. Through software failure time data analysis, we assessed the performance of the proposed model and identified new properties.

The results of this study are as follows.
First, an analysis of the reference data (MSE, R^2) for selecting an efficient model revealed that the Lomax and Goel-Okumoto models exhibited high levels of goodness-of-fit.
Second, an analysis of the attribute data (m(t), λ(t)) related to model performance showed that the Lomax model demonstrated superior true value prediction accuracy and a low error rate, making it the most efficient model.
Third, an analysis of the future reliability $\hat{R}(\tau)$ of the models indicated that while the proposed models tended to show a decline in reliability over time, the Pareto model maintained relatively higher reliability, confirming its effectiveness. Accordingly, among the proposed models, the Lomax distribution model was identified as the optimal model.

As a result, this study proposes a methodological solution process and key design data to help developers evaluate and predict attribute data in the early stages of software development. Furthermore, future research should aim to identify an industry-specific optimized model and explore additional reliability performance attributes based on the findings of this study.

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